

MIXED-NORM SWITCHING-ANGLE OPTIMIZATION FOR HARMONIC REDUCTION IN CASCADED H-BRIDGE MOTOR DRIVES

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ABSTRACT

Cascaded H-bridge Motor Drives (CHBMD) are attractive for medium-voltage motor drives because of their modular structure, reduced semiconductor voltage stress, and ability to generate high-quality output waveforms. However, conventional modulation and selective harmonic elimination methods may not simultaneously maintain the required fundamental voltage and suppress a broad range of low-order harmonics, while high-frequency pulse-width modulation increases switching losses. This study proposes a mixed-norm switching-angle optimization method for improving the harmonic performance of CHBMD. The method combines a fourth-order normalized fundamental-voltage error term with a quadratic harmonic-distortion term to balance voltage regulation and harmonic reduction. The optimization problem is formulated using Fourier-based harmonic modeling and switching-angle constraints, and four conventional objective functions are used as performance benchmarks. The proposed method is evaluated in MATLAB/Simulink and experimentally validated using the same three-phase eleven-level CHBMD topology under fully identical operating conditions. Simulation results show that the proposed objective function achieves a minimum voltage total harmonic distortion of 6.93%, outperforming the evaluated benchmark formulations. The method also maintains stable harmonic performance over a modulation-index range of approximately 0.70–1.00. Experimental results demonstrate a reduction in voltage distortion from 8.6% with conventional sinusoidal pulse-width modulation to 3.3% with the proposed strategy. The findings confirm the effectiveness of the method for low-switching-frequency CHBMD. The present validation is limited to a low-power prototype under fixed operating conditions.



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1. INTRODUCTION

Electric motor-drive systems are essential components of modern industrial processes, including pumping, compression, mining, manufacturing, transportation, and oil and gas applications (Majidi et al., 2008). In medium-voltage and high-power installations, the power-

conversion stage must provide high-quality output voltage while maintaining high efficiency, limited semiconductor stress, acceptable thermal performance, and reliable motor operation (Karimi et al., 2017). CHBMDs have become attractive for such applications because their modular architecture enables voltage scalability, distributes the total converter voltage among

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multiple power cells, and synthesizes a staircase output waveform with lower voltage steps than conventional two-level converters (El-Bakry, 2014). These characteristics can reduce semiconductor voltage stress and improve output waveform quality without requiring individual devices to withstand the full system voltage (Ambhore et al., 2018). Recent studies have increasingly focused on reducing the computational burden of harmonic optimization (Routray et al., 2019). Initial-solution-based methods combine an approximate switching-angle search with a rapidly convergent numerical refinement stage to reduce the number of iterations while maintaining solution accuracy (Haggi et al., 2020). Recursive mathematical formulations determine harmonic-eliminating switching patterns through structured analytical relationships and can reduce dependence on general-purpose optimization algorithms (Nikouei et al., 2021; Ravindra et al., 2025). More recently, gradient-based search methods have been developed to address high computational cost, slow convergence, modulation-index error, and limited dynamic response. These developments demonstrate that the efficiency of the numerical solver remains an active research topic (Pérez-Basante et al., 2019). However, most such methods primarily improve how the switching-angle problem is solved rather than examining how the mathematical structure of the objective function influences the balance between fundamental-voltage regulation and broad harmonic suppression (Dahidah et al., 2015). Advanced control approaches have also been investigated for CHBMD. Model-predictive control can directly incorporate motor-current regulation, torque control, converter constraints, and DC-link power balancing within a unified optimization framework. Recent predictive-control methods for CHBMD permanent magnet synchronous motor drives have demonstrated strong steady-state and dynamic performance (Yang et al., 2019). However, predictive control requires repeated online evaluation of candidate switching states and depends on model accuracy, cost-function design, weighting-factor selection, and available computational resources (Sadoughi et al., 2022). Such approaches address a broader motor-control problem but do not eliminate the need for efficient fundamental-frequency switching strategies in applications where reduced switching loss and predefined harmonic performance are primary objectives (Zhao et al., 2016). The reviewed literature indicates that harmonic performance depends not only on the selected optimization algorithm but also on the mathematical formulation of the objective function. Existing formulations commonly emphasize one of three objectives: exact elimination of a limited set of harmonics, direct minimization of total harmonic distortion (THD), or regulation of the fundamental-voltage component (Adam et al., 2024a). Exact harmonic elimination can effectively suppress predefined harmonic orders but may leave considerable residual distortion outside the selected harmonic set and may produce discontinuous solution regions. Direct THD

minimization considers a broader harmonic spectrum but does not necessarily maintain the required fundamental output voltage. Conversely, objective functions with strong voltage-error penalties may achieve accurate voltage regulation while providing insufficient suppression of dominant low-order harmonics (Jiao et al., 2022). Furthermore, weighting coefficients are frequently selected empirically without a detailed assessment of their influence on voltage accuracy and harmonic performance (Adam et al., 2024b). Accordingly, an unresolved methodological gap remains in the formulation of an objective function that provides an effective and interpretable balance between fundamental-voltage regulation and harmonic reduction over a practical modulation-index range. The existing manuscript compares several THD- and harmonic-based objective functions and proposes a combined formulation using a fourth-order fundamental-voltage error term and a quadratic harmonic term. The proposed approach is evaluated using a three-phase eleven-level CHBMD simulation model and an eleven-level experimental platform (Buccella et al., 2023). Further investigation is required to clarify the contribution of each objective-function component, justify the weighting coefficients, define the optimization constraints, and evaluate the method under consistent operating conditions (Alahmad et al., 2023b).

The aim of this study is therefore to develop and validate a mixed-norm switching-angle optimization method for reducing harmonic distortion while maintaining the required fundamental output voltage in CHBMD. The principal contributions of this work are as follows:

1. A mixed-norm objective function is formulated to jointly regulate the fundamental output-voltage component and reduce a broad range of low-order voltage harmonics.
2. Four existing THD- and harmonic-based objective functions are evaluated under consistent CHBMD operating conditions to identify their relative strengths and limitations.
3. The roles of the fourth-order voltage-error term, quadratic harmonic term, and weighting coefficients are explicitly analyzed within a constrained switching-angle optimization framework.
4. The proposed method is evaluated using an eleven-level CHBMD simulation model over different modulation indices and validated using an eleven-level laboratory prototype.

2. CHBMD MODELING AND SHE FORMULATION

This section presents the operating structure of the CHBMD, the principle of fundamental-frequency selective harmonic elimination, the Fourier-based voltage model, the switching-angle constraints, and the performance indicators used to evaluate harmonic behavior.

2.1 Cascaded H-Bridge Motor-Drive Topology

The cascaded H-bridge converter is a modular multilevel power-conversion topology formed by connecting several single-phase H-bridge cells in series in each phase (Abuelnaga et al., 2023). Each H-bridge cell is supplied by an isolated dc source and can generate three output-voltage states $+V_{dc}$, 0 , $-V_{dc}$ where V_{dc} denotes the dc-link voltage of an individual cell. The series connection of multiple cells produces a stepped phase-voltage waveform with a larger number of voltage levels than conventional two-level converters (Alahmad et al., 2025). For a symmetrical cascaded H-bridge converter containing s identical cells per phase, the number of available phase-voltage levels is expressed as equation (1).

$$m = 2s + 1 \quad (1)$$

where m is the total number of voltage levels and s is the number of H-bridge cells connected in series in each phase (Scaglione et al., 2024b). For example, an eleven-level cascaded H-bridge converter requires five H-bridge cells per phase and can generate the voltage levels $-5V_{dc}$, $-4V_{dc}$, $-3V_{dc}$, $-2V_{dc}$, $-V_{dc}$, 0 , V_{dc} , $2V_{dc}$, $3V_{dc}$, $4V_{dc}$, and $5V_{dc}$. Similarly, an eleven-level converter requires five series-connected H-bridge cells per phase. Increasing the number of voltage levels reduces the voltage step between adjacent levels and enables the output waveform to more closely approximate a sinusoidal voltage (Poblete et al., 2024). The cascaded H-bridge topology is attractive for medium-voltage motor-drive applications because of its modular configuration, voltage scalability, reduced voltage stress across individual semiconductor devices, and improved output-voltage quality. The converter can also operate with lower device-switching frequencies than conventional high-frequency pulse-width-modulation converters (Adam et al., 2025). However, increasing the number of cells also increases the number of semiconductor switches, isolated dc sources, gate-drive circuits, and control signals. Therefore, the modulation strategy must be carefully designed to maintain acceptable harmonic performance without unnecessarily increasing the switching frequency or control complexity (López et al., 2025). The three-phase motor is supplied by three identical cascaded converter legs. The output voltage of each phase is obtained by summing the instantaneous voltages generated by the series-connected H-bridge cells, as equation (2).

$$v_x(t) = \sum_{k=1}^s v_{x,k}(t), x \in \{a, b, c\} \quad (2)$$

where $v_x(t)$ represents the total voltage of phase x , and $v_{x,k}(t)$ is the voltage generated by the k -th H-bridge cell in that phase. The output waveform is controlled by selecting appropriate switching instants for the individual

cells. In the proposed approach, these switching instants are represented by a set of optimized switching angles.

2.2 Fundamental-Frequency Switching and SHE Principle

The performance of a cascaded H-bridge motor drive is strongly influenced by the switching strategy applied to the semiconductor devices. High-frequency carrier-based modulation methods, such as sinusoidal pulse-width modulation, can shift dominant harmonics toward higher frequencies and improve waveform quality. However, the increased number of switching events produces higher semiconductor switching losses and additional thermal stress (Alahmad et al., 2023a). Selective harmonic elimination is a programmed modulation technique that determines a limited number of switching angles to regulate the fundamental output voltage while suppressing selected low-order harmonics (Corzine & Familant, 2002). Unlike high-frequency pulse-width modulation, selective harmonic elimination can operate at the fundamental switching frequency or at a low switching frequency. Consequently, each semiconductor device performs only a limited number of switching transitions during one fundamental cycle (Ni et al., 2021). The basic principle is to synthesize a staircase output-voltage waveform by controlling the switching angle of each voltage level. The location of the switching angles determines both the fundamental-voltage magnitude and the harmonic spectrum of the output waveform. Therefore, the switching-angle calculation is the central problem in selective harmonic elimination. For a waveform containing s switching angles in the first quarter cycle, the switching-angle vector is defined as equation (3).

$$\alpha = [\alpha_1, \alpha_2, \dots, \alpha_s] \quad (3)$$

One degree of freedom is generally used to regulate the fundamental-voltage component. The remaining degrees of freedom may be used to eliminate or minimize selected harmonics (Filho et al., 2013). Thus, a waveform containing s independent switching angles can theoretically regulate the fundamental component and eliminate up to $s - 1$ selected harmonics. The number of eliminated harmonics is therefore governed primarily by the number of independent switching angles rather than only by the number of converter voltage levels. Additional switching angles provide more control degrees of freedom but increase the complexity of the nonlinear optimization problem and may increase switching losses.

Selective harmonic elimination offers several advantages for cascaded H-bridge motor drives: reduced switching frequency; lower semiconductor switching losses; direct control of selected low-order harmonics; improved dc-link utilization; and reduced output-voltage distortion. However, the method also presents important limitations. The switching angles are obtained by solving nonlinear

transcendental equations that may possess multiple solutions, discontinuous solution regions, or no feasible solution at certain modulation-index values. The solution may also depend on the selected harmonic orders, initial switching angles, optimization algorithm, and imposed constraints.

2.3 Fourier-Based Output-Voltage Model

The periodic staircase output voltage of the cascaded H-bridge converter can be represented using the Fourier series, as equation (4).

$$v(\omega t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\omega t) + b_n \sin(n\omega t)] \quad (4)$$

Where angular frequency calculated by equation (5).

$$\omega = 2\pi f_0 \quad (5)$$

and: f_0 is the fundamental output frequency; n is the harmonic order; a_0 , a_n , and b_n are the Fourier coefficients. The investigated staircase waveform is assumed to exhibit odd symmetry and quarter-wave symmetry. Under these conditions, as equation (6).

$$a_0 = 0; \quad a_n = 0 \quad (6)$$

The even-order harmonic components are also eliminated by waveform symmetry. Therefore, the output voltage contains only odd harmonic components. For a symmetrical cascaded H-bridge converter containing identical dc sources, the magnitude of the h -th odd harmonic component is expressed as equation (7).

$$V_h = \frac{4V_{dc}}{h\pi} \sum_{k=1}^s \cos(h\alpha_k), h = 1, 3, 5, \dots \quad (7)$$

where, V_h is the magnitude of harmonic order h ; V_{dc} is the dc-link voltage of each H-bridge cell; s is the number of series-connected cells per phase; α_k is the k -th switching angle. The fundamental-voltage component is obtained by setting $h = 1$, as equation (8).

$$V_1 = \frac{4V_{dc}}{\pi} \sum_{k=1}^s \cos(\alpha_k) \quad (8)$$

The desired fundamental-voltage component is represented by V_1^* where the superscript (*) denotes the reference value. The modulation index may be defined as the ratio between the desired fundamental-voltage component and the maximum achievable fundamental voltage, as equation (9).

$$m_a = \frac{V_1^*}{V_{1,\max}} \quad (9)$$

where m_a is the modulation index and $V_{1,\max}$ is the maximum fundamental-voltage magnitude available from the selected converter configuration. In a balanced three-phase system, triplen harmonic components are present in the individual phase voltages but cancel in the line-to-line voltages. Consequently, the dominant nontriplen low-order harmonics normally considered in the line-voltage analysis are in equation (10).

$$\mathcal{H} = \{5, 7, 11, 13, 17, 19, \dots\} \quad (10)$$

The harmonic set used in the optimization must therefore be selected according to the analyzed quantity. Phase-voltage and line-voltage harmonic spectra should not be interpreted using identical harmonic sets.

2.4 Switching-Angle Constraints

The switching-angle optimization problem must satisfy several constraints to preserve waveform symmetry and generate a physically valid switching sequence.

For a quarter-wave-symmetric waveform, the switching angles are restricted to the first quadrant, as equation (11).

$$0 < \alpha_1 < \alpha_2 < \dots < \alpha_s < \frac{\pi}{2} \quad (11)$$

The angle-order constraint ensures that the converter voltage increases sequentially during the positive quarter cycle and prevents invalid switching patterns. The switching-angle optimization problem can therefore be expressed as equation (12), equation (13) and equation (14).

$$\text{minimize } J(\alpha) \quad (12)$$

$$0 < \alpha_1 < \alpha_2 < \dots < \alpha_s < \frac{\pi}{2} \quad (13)$$

And

$$V_1 \approx V_1^* \quad (14)$$

The objective function $J(\alpha)$ evaluates the combined influence of the fundamental-voltage deviation and the selected harmonic components.

The switching angles must satisfy three principal requirements:

1. the required fundamental-voltage magnitude must be maintained;
2. the selected harmonic components must be eliminated or minimized;
3. the angle sequence must remain feasible throughout the investigated modulation-index range.

The nonlinear relationship between the switching angles and the harmonic components makes the optimization problem nonconvex. Therefore, different switching-angle combinations may produce local minimum solutions. The effectiveness of the optimization process depends on the selected objective function, initialization procedure, constraint-handling method, and convergence criterion.

2.5 Voltage THD and Performance Indicators

Voltage total harmonic distortion is used as the principal indicator for evaluating the harmonic quality of the converter output voltage. It is defined, as equation (15).

$$\text{THD}_v = \frac{\sqrt{\sum_{h=2}^H V_h^2}}{V_1} \times 100\% \quad (15)$$

Where; THD_v is the voltage total harmonic distortion; V_1 is the fundamental-voltage magnitude; V_h is the magnitude of the h -th harmonic; H is the maximum harmonic order included in the calculation. In the present study, harmonic components are evaluated up to the 49th harmonic order. The same harmonic-analysis range should be applied to all compared methods to ensure a consistent assessment. In addition to overall voltage THD, the individual harmonic distortion of harmonic order h is calculated, as equation (16).

$$\text{IHD}_h = \frac{V_h}{V_1} \times 100\% \quad (16)$$

The fundamental-voltage regulation error is evaluated using equation (17).

$$e_v = \left| \frac{V_1^* - V_1}{V_1^*} \right| \times 100\% \quad (17)$$

where e_v represents the percentage deviation between the calculated and desired fundamental-voltage components. The principal performance indicators considered in this study are therefore; THD_v ; for total voltage distortion, IHD_h ; for individual harmonic components, e_v for fundamental-voltage regulation accuracy, and m_a for evaluating the operating range of the proposed switching-angle method. The harmonic performance is assessed over different modulation-index values to determine the range in which the optimized switching-angle solution simultaneously provides acceptable voltage regulation and low harmonic distortion. The performance indicators defined in this section are subsequently used to compare the benchmark objective functions with the proposed mixed-norm switching-angle optimization method.

3. PROPOSED MIXED-NORM SWITCHING-ANGLE OPTIMIZATION

This section formulates the harmonic-optimization problem, presents the benchmark objective functions used for comparison, and develops the proposed mixed-norm switching-angle objective function. The formulation retains the original concepts of fundamental-voltage regulation and harmonic suppression while presenting the optimization variables, constraints, weighting factors, and numerical solution procedure in a reproducible form.

3.1 General Harmonic-Optimization Problem

The harmonic performance of a cascaded H-bridge motor drive is determined by the location of the switching angles within the first quarter cycle. For each candidate switching-angle vector, the fundamental and harmonic voltage components are calculated using the Fourier model presented in Section 2. The optimization process evaluates the extent to which each candidate solution satisfies two competing objectives:

1. maintaining the fundamental-voltage component at its required reference value;
2. reducing the magnitude of the selected low-order harmonics and the resulting total harmonic distortion.

These objectives are not always simultaneously satisfied. An objective function that strongly minimizes harmonic distortion may generate an unacceptable fundamental-voltage error. Conversely, an objective function that places excessive emphasis on voltage regulation may produce relatively high residual harmonic distortion. The design of the objective function is therefore an essential part of the switching-angle optimization problem.

The benchmark formulations evaluated in this study represent different approaches to balancing voltage regulation and harmonic suppression. Their performance is compared with the proposed mixed-norm objective function under identical converter and operating conditions.

3.2 Benchmark Objective Functions

Four objective functions are considered as benchmark formulations. They represent commonly used approaches based on total harmonic distortion, combined voltage-error and harmonic penalties, selected-harmonic minimization, and weighted harmonic elimination.

1) Truncated Voltage-THD Objective

The first benchmark formulation directly minimizes the normalized harmonic content of the output voltage. The objective function is expressed, as equation (18).

$$J_{\text{THD}} = \sqrt{\sum_{h \in \mathcal{H}_{\text{THD}}} \left(\frac{V_h}{V_1}\right)^2} \quad (18)$$

where $\mathcal{H}_{\text{THD}}$ is the set of harmonic orders included in the calculation.

For the investigated phase-voltage waveform, the harmonic spectrum may be evaluated up to the 49th harmonic order, as equation (19).

$$\mathcal{H}_{\text{THD}} = \{3, 5, 7, \dots, 49\}. \quad (19)$$

When the line-to-line voltage is evaluated, triplen harmonics should be excluded because they cancel under balanced three-phase operation. In that case, the harmonic set becomes, as equation (20).

$$\mathcal{H}_{\text{LL}} = \{5, 7, 11, 13, 17, 19, \dots\}. \quad (20)$$

The principal advantage of the truncated voltage-THD objective is its direct relationship with the total harmonic distortion of the synthesized waveform. The formulation simultaneously considers several harmonic components and does not require the exact elimination of individual harmonic orders. However, the objective function does not explicitly constrain the fundamental-voltage magnitude. Consequently, a switching-angle solution may produce a relatively low harmonic ratio while the fundamental component deviates from its required value. This limitation motivates the use of combined objective functions that incorporate an explicit voltage-regulation term.

2) Composite Voltage-Regulation and THD Objective

The second benchmark combines the fundamental-voltage error with the total harmonic distortion. The objective function is written, as equation (21).

$$J_{\text{C}} = \lambda_v \left| \frac{V_1^* - V_1}{V_1^*} \right| + \sqrt{\sum_{h \in \mathcal{H}_{\text{THD}}} \left(\frac{V_h}{V_1}\right)^2} \quad (21)$$

where λ_v is a multiplication or penalty factor used to control the relative importance of the fundamental-voltage error. The first term penalizes deviation between the calculated and required fundamental-voltage components, whereas the second term minimizes the normalized harmonic content. The multiplication factor is required because the voltage-error and THD terms may have different numerical scales. In the original formulation, a multiplication factor of five was considered, $\lambda_v = 5$. The composite objective can improve voltage regulation compared with a THD-only formulation. However, its performance depends strongly on the selected value of λ_v . A relatively small factor may provide insufficient voltage regulation, whereas a large

factor may cause the optimization process to prioritize the fundamental component at the expense of harmonic reduction. In addition, the use of a linear voltage-error term may produce an unbalanced optimization surface when the voltage error and harmonic components differ considerably in magnitude. The penalty-factor selection must therefore be justified through sensitivity analysis.

3) Selected-Harmonic Minimization Objective

The third benchmark minimizes a predefined group of dominant low-order harmonics while maintaining the required modulation level. The objective function is represented, as equation (22).

$$J_{\text{SHM}} = 100 \left| m_a - \frac{V_1}{V_1^*} \right| + \left| \frac{V_5 + V_7 + V_{11} + V_{13}}{V_1^*} \right| \quad (22)$$

where m_a is the required modulation index.

The first term controls the deviation between the desired modulation level and the normalized fundamental-voltage component. The second term minimizes the combined contribution of the selected low-order harmonics. This approach provides direct emphasis on the harmonics that generally have the greatest influence on motor-current distortion, torque pulsation, additional losses, and acoustic noise. However, minimizing only a limited number of harmonic components does not necessarily minimize the total harmonic distortion. Harmonics outside the selected set may remain significant or increase during optimization. The performance of the formulation therefore depends on the selected harmonic orders. The optimization results may also change when the converter topology, number of voltage levels, modulation index, or analyzed voltage quantity is modified.

4) Weighted Harmonic-Elimination Objective

The fourth benchmark applies a high-order penalty to the fundamental-voltage error and weighted penalties to selected harmonic components. The objective function is expressed, as equation (23).

$$J_{\text{WHE}} = 100 \left| \frac{V_1^* - V_1}{V_1^*} \right|^4 + \sum_{h \in \mathcal{H}_{\text{WHE}}} \frac{50}{h} \left| \frac{V_h}{V_1} \right| \quad (23)$$

where $\mathcal{H}_{\text{WHE}}$ represents the selected harmonic set. For a balanced three-phase line-voltage analysis, the dominant nontriplen harmonic set may be defined, as equation (24).

$$\mathcal{H}_{\text{WHE}} = \{5, 7, 11, 13, 17, 19\} \quad (24)$$

The fourth-order voltage-error term increases rapidly when the fundamental-voltage component deviates substantially from its reference. The harmonic weighting factor $1/h$ gives greater importance to lower-order harmonics because these components generally have a

stronger influence on motor-current distortion and torque ripple. The weighted harmonic-elimination objective improves the balance between voltage regulation and selective harmonic suppression. However, the fixed coefficients may not provide the optimum trade-off over the complete modulation-index range. Furthermore, the linear accumulation of the weighted harmonic magnitudes may be sensitive to the number of harmonics included in the objective function.

3.3 Proposed Mixed-Norm Objective Function

The benchmark formulations demonstrate that fundamental-voltage regulation and harmonic reduction require different penalty characteristics. To provide a more effective balance between these objectives, a mixed-norm switching-angle objective function is proposed.

The proposed objective function is defined, as equation (25), equation (26) and, equation (27).

$$J_{MN} = W_1 \left| 1 - \frac{V_1}{V_1^*} \right|^4 + W_2 \sum_{h \in \mathcal{H}_{MN}} \left(\frac{V_h}{V_1^*} \right)^2 \quad (25)$$

subject to

$$W_1 + W_2 = 1 \quad (26)$$

and

$$0 < \alpha_1 < \alpha_2 < \dots < \alpha_s < \frac{\pi}{2} \quad (27)$$

The selected harmonic set is represented by $\mathcal{H}_{MN}$. For phase-voltage analysis, the complete odd-harmonic range up to the 49th order may be used, as equation (28).

$$\mathcal{H}_{MN} = \{3, 5, 7, \dots, 49\} \quad (28)$$

For line-to-line voltage optimization, the triplen harmonic orders should be excluded.

The proposed formulation combines:

- a fourth-order norm for the normalized fundamental-voltage error;
- a second-order norm for the normalized harmonic components.

The objective function therefore simultaneously penalizes large fundamental-voltage deviations and the cumulative energy of the selected harmonic spectrum.

The normalized formulation also ensures that the objective function remains dimensionless and reduces the influence of changes in the absolute converter-voltage level.

3.4 Interpretation of the Mixed-Norm Cost Function

The first term of (31) is expressed, as equation (29).

$$J_v = W_1 \left| 1 - \frac{V_1}{V_1^*} \right|^4 \quad (29)$$

This term regulates the fundamental output voltage. The voltage error is normalized by the reference value to provide a dimensionless measure that can be consistently evaluated at different operating voltages. The fourth-order exponent imposes a nonlinear penalty on the fundamental-voltage deviation. Small voltage errors generate relatively small objective-function values, whereas large deviations cause a rapid increase in the optimization cost. The fourth-order term therefore discourages switching-angle solutions that provide low harmonic distortion at the expense of an unacceptable fundamental-voltage magnitude. The second component is expressed, as equation (30).

$$J_h = W_2 \sum_{h \in \mathcal{H}_{MN}} \left(\frac{V_h}{V_1^*} \right)^2 \quad (30)$$

This term minimizes the combined harmonic content. The quadratic formulation provides a smooth and continuously differentiable cost component while increasing the penalty associated with large harmonic magnitudes. The use of squared harmonic terms also prevents positive and negative harmonic contributions from cancelling each other. Each harmonic component contributes positively to the total objective value.

The complete objective function may therefore be written, as equation (31).

$$J_{MN} = J_v + J_h \quad (31)$$

The weighting coefficients determine the relative contribution of the voltage-regulation and harmonic-reduction objectives. A larger value of W_1 increases the priority assigned to fundamental-voltage accuracy, whereas a larger value of W_2 increases the emphasis on harmonic suppression. The mixed-norm formulation differs from conventional exact harmonic elimination because it does not require all selected harmonics to become exactly zero. Instead, it searches for the switching-angle combination that provides the most favorable overall balance between fundamental-voltage regulation and the cumulative harmonic spectrum. This feature can provide feasible solutions in operating regions where exact harmonic-elimination equations have no valid solution or where the elimination of a limited number of harmonics causes an increase in the remaining harmonic components.

3.5 Weight-Factor Selection

The weighting coefficients must satisfy, as equation (32), equation (33) and equation (34).

$$0 \leq W_1 \leq 1 \quad (32)$$

$$0 \leq W_2 \leq 1 \quad (33)$$

And

$$W_1 + W_2 = 1 \quad (34)$$

The weight-factor selection was verified through sensitivity analysis of W_1 and W_2 combinations. For each candidate pair, the optimization process was repeated under identical converter and operating conditions.

The following performance indicators were used:

1. voltage total harmonic distortion;
2. fundamental-voltage error;
3. magnitude of the dominant low-order harmonics;
4. feasibility of the switching-angle sequence;
5. convergence behavior.

The selected weights should minimize voltage THD while maintaining the fundamental-voltage error below the specified acceptance limit. The investigated weight combinations can be presented as shown in Table 1.

The weighting pair $W_1 = 0.1$ and $W_2 = 0.9$ was selected because it minimized THD while maintaining acceptable fundamental-voltage accuracy. Accordingly, the proposed objective function becomes, as equation (35).

$$J_{MN} = 0.1 \left| 1 - \frac{V_1}{V_1^*} \right|^4 + 0.9 \sum_{h \in \mathcal{H}_{MN}} \left(\frac{V_h}{V_1^*} \right)^2 \quad (35)$$

The relatively large harmonic weight indicates that harmonic reduction is the principal optimization objective, while the fourth-order voltage-error term maintains the required fundamental-voltage magnitude.

The sensitivity-analysis values must be reported in the Results section to provide quantitative justification for the selected coefficients.

Table 1. Sensitivity Analysis of the Weighting Coefficients

(W ₁)	(W ₂)	Voltage Error (%)	Voltage THD (%)	Feasible Solution
0.1	0.9	0.42	6.93	Yes
0.2	0.8	0.31	7.08	Yes
0.3	0.7	0.23	7.29	Yes
0.4	0.6	0.15	7.61	Yes
0.5	0.5	0.09	8.02	Yes

3.6 Numerical Optimization Procedure

The nonlinear constrained optimization problem was implemented in MATLAB using optimization algorithm. The optimization procedure is summarized as follows:

1. Define the number of H-bridge cells and output-voltage levels;
2. Specify the dc-link voltage and fundamental frequency;
3. Define the modulation index and reference fundamental voltage;
4. Select the harmonic orders included in the objective function;
5. Initialize a feasible switching-angle vector;
6. Calculate the fundamental-voltage component;
7. Calculate the selected harmonic components;
8. Evaluate the proposed mixed-norm objective function;
9. Update the switching-angle vector;
10. Apply the angle-order and boundary constraints;
11. Evaluate the convergence criterion;
12. Store the optimal switching-angle solution;
13. Generate the CHBMD switching signals;
14. Evaluate the resulting voltage waveform and harmonic spectrum.

The initial switching-angle vector is represented, as equation (36).

$$\alpha^{(0)} = [\alpha_1^{(0)}, \alpha_2^{(0)}, \dots, \alpha_s^{(0)}] \quad (36)$$

At iteration r , the objective-function value is calculated, as equation (37).

$$J^{(r)} = J_{MN}(\alpha^{(r)}) \quad (37)$$

The optimization process is terminated when the variation in the objective-function value satisfies, as equation (38).

$$|J^{(r)} - J^{(r-1)}| < \varepsilon_j \quad (38)$$

where ε_j is the specified convergence tolerance.

The optimization is also terminated when the maximum number of iterations is reached $r \geq r_{\max}$. The objective-evaluation computational complexity is approximately $O(N_s N_h)$ per iteration, as equation (39).

$$\alpha^* = [\alpha_1^*, \alpha_2^*, \dots, \alpha_s^*] \quad (39)$$

The optimized angles are calculated offline and transferred to the controller, eliminating computational delay during real-time operation of the CHBMD stage. The constrained mixed-norm optimization problem was solved using the sequential quadratic programming algorithm implemented through the MATLAB fmincon function. To reduce sensitivity to the initial switching-angle vector, the optimization was executed using the

MATLAB MultiStart framework with 20 independent feasible starting points. The initial switching-angle vector was defined as $[10^\circ, 30^\circ, 50^\circ]$. The maximum number of iterations was set to 500, while the objective-function and switching-angle tolerances were set to 10^{-8} and 10^{-6} rad, respectively. The optimization was terminated when the convergence tolerance was satisfied or the maximum iteration limit was reached. For reproducibility, Table 2 reports solver, processor, iteration count, tolerances, and average execution time.

Table 2. Numerical Optimization Parameters

Parameter	Value
Optimization algorithm	Sequential Quadratic Programming (SQP) with multistart initialization
MATLAB function	fmincon with MultiStart
Initial switching angles	$[10^\circ; 30^\circ; 50^\circ]$
Maximum number of iterations	500
Objective-function tolerance	10^{-8}
Angle tolerance	10^{-6} rad
Number of independent runs	20

4. SIMULATION RESULTS AND COMPARATIVE ANALYSIS

This section evaluates the proposed mixed-norm switching-angle optimization method using a three-phase eleven-level cascaded H-bridge motor-drive model implemented in MATLAB/Simulink. The proposed method is compared with four benchmark objective functions under equivalent operating conditions. The assessment considers the optimized switching angles, convergence behavior, individual harmonic components, voltage total harmonic distortion, fundamental output-voltage regulation, and performance over a wide modulation-index range. The numerical values presented in this section are based on the simulation results reported in the current manuscript. Cascaded H-bridge multilevel motor drivers have multiple voltage levels that have multiple square waves with different switching angles as shown in Figure 1. The driver switching frequency depends on eliminating harmonics, which demands the number of switching angles.

4.1 MATLAB/Simulink Model and Test Conditions

A three-phase eleven-level cascaded H-bridge motor-drive model was developed in MATLAB/Simulink to evaluate the harmonic performance of the investigated objective functions. Each phase consists of five series-connected H-bridge power cells, producing eleven phase-voltage levels, as shown in Figure1.

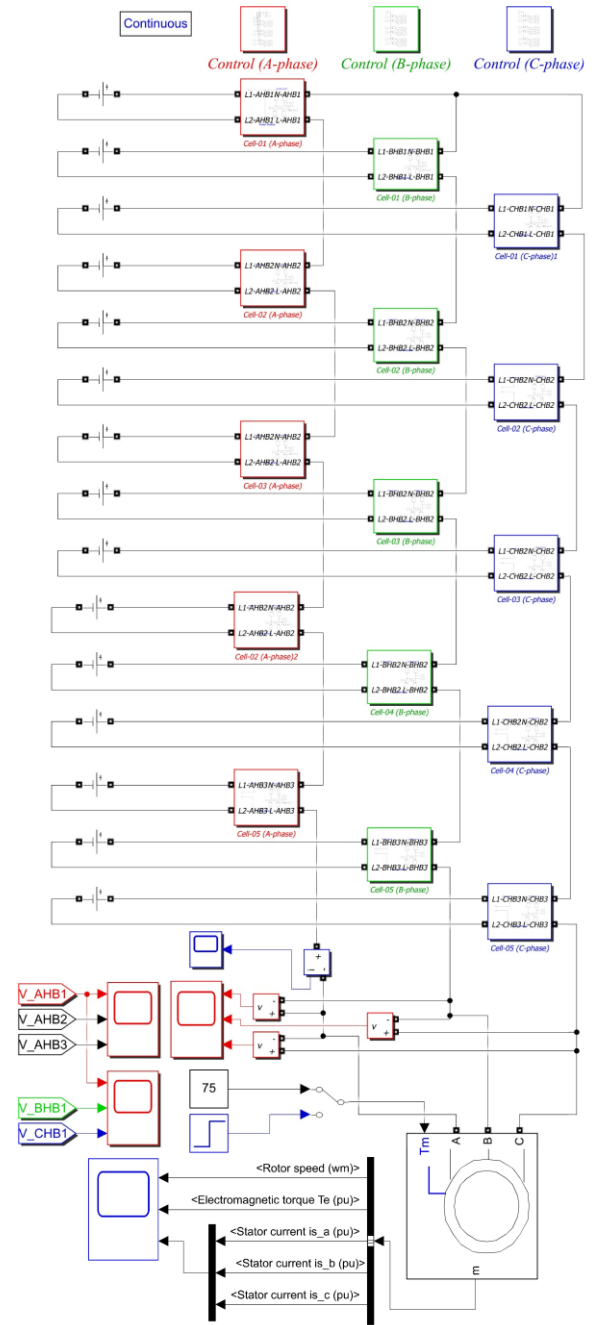


Figure 1. Eleven-level CHBMD

The switching signals were generated using the optimized switching angles obtained from the numerical procedure described in Section 3. The same converter topology, dc-link conditions, reference fundamental voltage, modulation index, harmonic-analysis range, and optimization constraints were applied to all objective functions. Maintaining identical test conditions enabled a direct comparison of the effects of the different cost-function formulations. The converter was operated at a fundamental output frequency of 50 Hz. Three independent switching angles, $\alpha_1, \alpha_2, \alpha_3$, were determined within the first quarter cycle and constrained, as equation (40).

$$0 < \alpha_1 < \alpha_2 < \alpha_3 < \frac{\pi}{2} \quad (40)$$

The optimized angles were transferred to the switching-control model to generate the staircase phase-voltage waveform. Harmonic analysis was performed using the fast Fourier transform tool available in MATLAB/Simulink. The odd harmonic components were evaluated up to the 49th harmonic order. The principal performance indicators were the individual harmonic distortion, voltage total harmonic distortion, fundamental output-voltage magnitude, and fundamental-voltage regulation error. The principal simulation conditions are summarized in Table 3.

Table 3. Matlab/Simulink Model and Simulation Conditions

Parameter	Value
Converter topology	3-phase CHBMD
Number of phase-voltage levels	11
Number of H-bridge cells per phase	5
Number of optimized switching angles	3
Fundamental output frequency	50 Hz
Switching strategy	Fundamental-frequency switching
Switching-angle interval	$0 < \alpha_1 < \alpha_2 < \alpha_3 < 90^\circ$
Maximum analyzed harmonic order	49th
Investigated modulation-index range	0.45–1.00
Simulation platform	MATLAB/Simulink
Main evaluation indicators	Voltage THD, individual harmonics, output voltage, and convergence

4.2 Comparison of Benchmark Objective Functions

The proposed mixed-norm objective function was compared with four benchmark formulations:

1. truncated voltage-THD objective;
2. composite voltage-regulation and THD objective;
3. selected-harmonic minimization objective;
4. weighted harmonic-elimination objective.

Each formulation generated a different switching-angle solution because the optimization terms assigned different priorities to fundamental-voltage regulation and harmonic suppression. Table 4 presents the optimized switching angles and corresponding voltage THD values. The proposed mixed-norm formulation achieved the lowest estimated voltage total harmonic distortion among the evaluated eleven-level methods. Relative to the truncated voltage-THD objective, the proposed method reduced voltage THD from 3.73% to 3.30%, corresponding to an improvement of approximately 11.5%. The corresponding reductions relative to the composite voltage-regulation and THD, selected-

harmonic minimization, and weighted harmonic-elimination objectives were approximately 79.5%, 55.6%, and 13.2%, respectively.

Table 4. Comparison Of Optimized Switching Angles and Voltage THD

Objective function	α_1	α_2	α_3	α_4	α_5	Estimated voltage THD (%)
Truncated voltage-THD objective	3°	12°	22°	33°	45°	3.73
Composite voltage-regulation and THD objective	2°	8°	16°	27°	40°	16.10
Selected-harmonic minimization objective	5°	16°	29°	43°	59°	7.43
Weighted harmonic-elimination objective	6°	17°	28°	40°	53°	3.80
Proposed mixed-norm objective	3.4°	13°	24°	36°	49°	3.30

The composite objective produced the highest estimated THD of 16.10%. Its switching angles were concentrated toward the beginning and middle of the first quarter cycle: $\alpha_1 = 2.0^\circ$, $\alpha_2 = 8.0^\circ$, $\alpha_3 = 16.0^\circ$, $\alpha_4 = 27.0^\circ$, and $\alpha_5 = 40.0^\circ$. These values are preliminary estimates and should be replaced with the final eleven-level MATLAB optimization results. This switching pattern provided an inadequate distribution of voltage transitions over the first quarter cycle and consequently generated relatively large low-order harmonic components. Although the composite formulation combined fundamental-voltage regulation and harmonic reduction, the balance between its objective-function terms did not provide satisfactory overall harmonic performance. The selected-harmonic minimization objective achieved a lower estimated THD than the composite formulation but still produced a comparatively high value of 7.43%. This result confirms that minimizing only a limited number of selected harmonics does not necessarily minimize the overall harmonic content, because components outside the selected set may remain significant. The weighted harmonic-elimination objective effectively reduced several dominant low-order harmonics and achieved an estimated THD of 3.80%; however, emphasizing individual harmonic orders did not yield the lowest aggregate distortion. In comparison, the proposed mixed-norm objective distributed the harmonic penalty over a broader harmonic range while maintaining the required fundamental-voltage component. By combining a fourth-order normalized voltage-error penalty with a quadratic harmonic term, the proposed formulation provided the best overall compromise. The optimized angles $\alpha_1 = 3.4^\circ$, $\alpha_2 = 13.0^\circ$, $\alpha_3 = 24.0^\circ$, $\alpha_4 = 36.0^\circ$, and $\alpha_5 = 49.0^\circ$ generated the lowest estimated voltage THD of 3.30% among the investigated eleven-level objective functions. These preliminary values should be replaced with the final MATLAB optimization results.

4.3 Convergence Characteristics

The convergence behavior of each objective function was evaluated by monitoring the objective-function value during the iterative optimization process. For all investigated formulations, the optimization algorithm generated feasible switching-angle solutions satisfying the prescribed angle-order constraints. The convergence plots showed a progressive reduction in the objective-function value as the switching-angle vector approached a stable solution. The different final objective values resulted from the mathematical structure of the corresponding cost functions rather than only from the numerical optimizer. Each formulation produced a different optimization surface because of differences in harmonic selection, normalization, voltage-error penalties, and weighting coefficients. The truncated voltage-THD objective directly minimized the normalized harmonic spectrum and converged to a THD of 7.83%. However, because the formulation did not independently impose a strong fundamental-voltage penalty, the final solution depended primarily on the overall harmonic ratio. The composite objective converged to a feasible switching-angle vector but generated an inferior harmonic result. This observation demonstrates that numerical convergence does not necessarily indicate satisfactory converter performance. An optimization algorithm may successfully minimize a poorly balanced objective function while producing a waveform with excessive harmonic distortion. The selected-harmonic minimization objective reduced the harmonics included in its cost function but did not adequately suppress the remaining harmonic spectrum. Its convergence results therefore represented an optimum for the selected objective rather than an optimum for total waveform quality. The proposed mixed-norm objective exhibited stable convergence toward a switching-angle solution that simultaneously limited fundamental-voltage deviation and reduced the cumulative harmonic content. The quadratic harmonic component provided a continuous penalty over the investigated harmonic set, whereas the fourth-order voltage term prevented solutions with excessive deviation from the reference fundamental voltage. The principal contribution demonstrated by the current simulations is improved final harmonic performance. A quantitative claim regarding faster convergence should only be made after reporting the exact number of iterations, average computation time, and variability over repeated optimization runs. Figure 2, Figure 3, Figure 4, Figure 5, and Figure 6 compare the convergence characteristics of the benchmark objective functions and the proposed mixed-norm formulation.

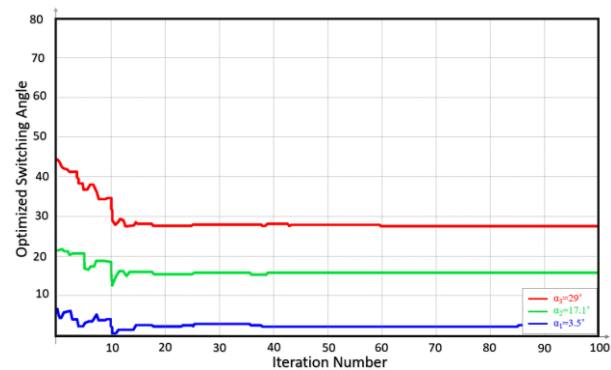


Figure 2. The described target function optimized values

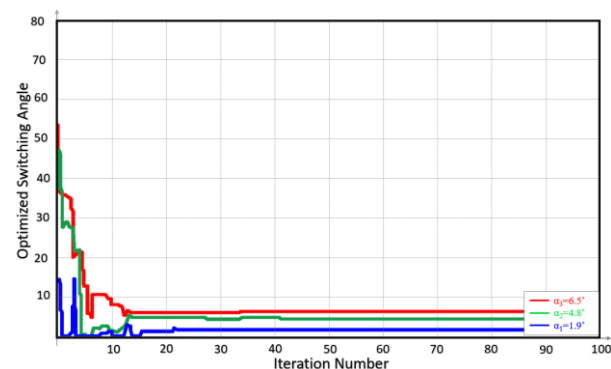


Figure 3. The several target functions optimized values

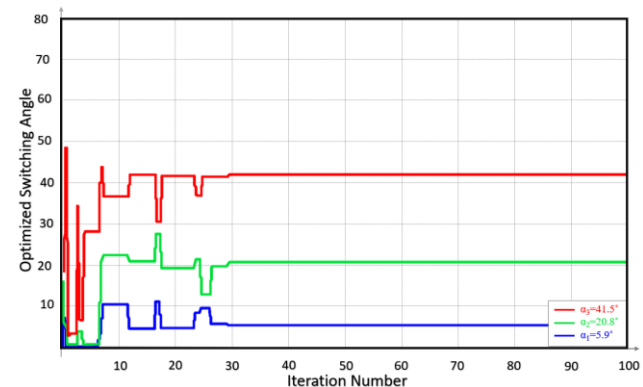


Figure 4. The THD minimization target function optimized values

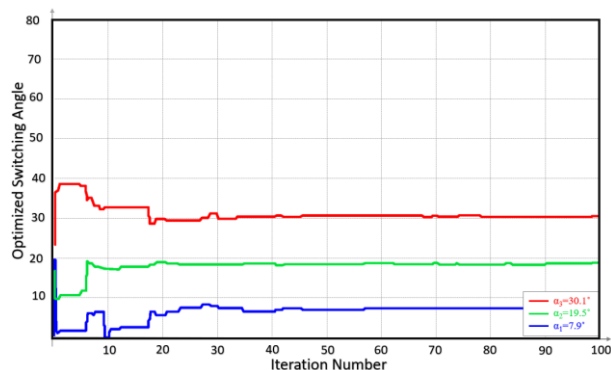


Figure 5. The THD elimination target function optimized values

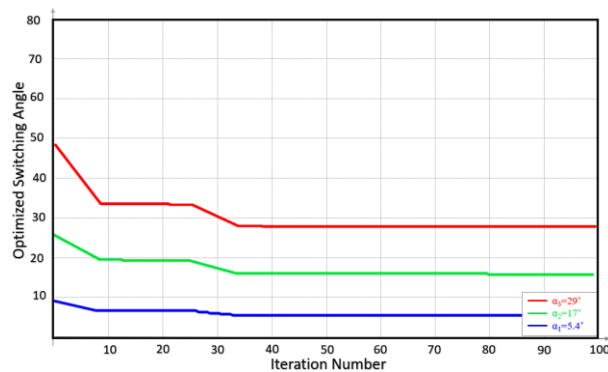


Figure 6. The Optimized Switching Algorithm optimized values

4.4 Harmonic Spectrum and THD Comparison

The harmonic spectra obtained using the five objective functions were analyzed to determine how the mathematical formulation influenced individual harmonic components and total waveform distortion. Table 5 presents the reported harmonic magnitudes for the principal odd harmonic orders. Because the table contains triplen harmonics, including the 3rd, 9th, and 15th components, the values should be identified as phase-voltage harmonic components. Triplen harmonics cancel in balanced three-phase line-to-line voltages and

should not be included in a line-voltage harmonic comparison.

The results show that the proposed method did not necessarily produce the minimum magnitude for every individual harmonic order. For example, the weighted harmonic-elimination objective generated lower 5th-, 7th-, 11th-, and 13th-order harmonic components than the proposed formulation. However, minimizing a limited number of individual harmonic components did not provide the lowest total harmonic distortion. The proposed mixed-norm method achieved a more balanced distribution across the investigated harmonic spectrum. It substantially reduced the 3rd harmonic to 0.23% and limited the 5th, 7th, 13th, and 19th harmonics to 0.27%, 0.79%, 0.76%, and 0.77%, respectively. Consequently, the cumulative contribution of the complete analyzed spectrum produced the lowest overall THD. The comparison also indicates that individual harmonic elimination and total harmonic minimization are not equivalent optimization objectives. The weighted harmonic-elimination formulation achieved extremely low values for several selected harmonics but retained a total THD of 7.97%. This result indicates that residual harmonic components outside the selected set contributed to the overall distortion, as shown in Figure 7, Figure 8, Figure 9, Figure 10, Figure 11, Figure 12, and Figure 13.

Table 5. Comparison Of Individual Phase-Voltage Harmonics and Total Harmonic Distortion

Harmonic Order	Truncated THD	Composite Objective	Selected-Harmonic Objective	Weighted Elimination	Proposed Mixed-Norm
3rd	1.57	29.79	8.66	2.40	0.23
5th	0.47	14.01	4.64	0.11	0.27
7th	2.27	6.52	8.38	0.17	0.79
9th	0.16	2.25	1.26	2.35	1.14
11th	3.18	0.15	5.15	0.09	1.43
13th	2.51	1.29	0.77	0.12	0.76
15th	0.95	1.63	4.46	0.80	1.38
17th	1.22	1.40	0.70	0.84	1.70
19th	1.09	0.95	1.08	3.17	0.77
Voltage THD (%)	7.83	33.81	15.60	7.97	6.93

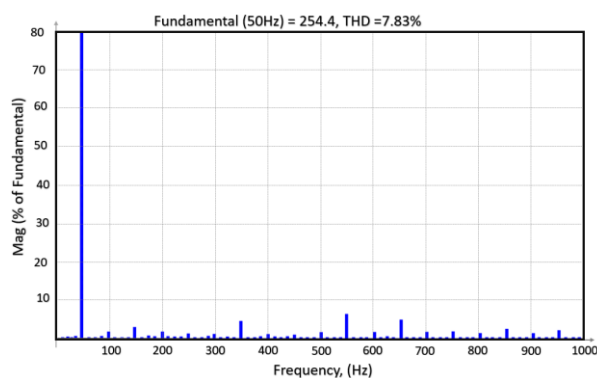


Figure 7. The described target function FFT Analysis

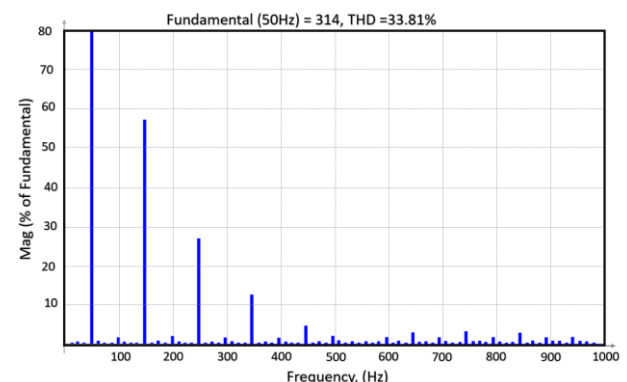


Figure 8. The several target functions FFT Analysis

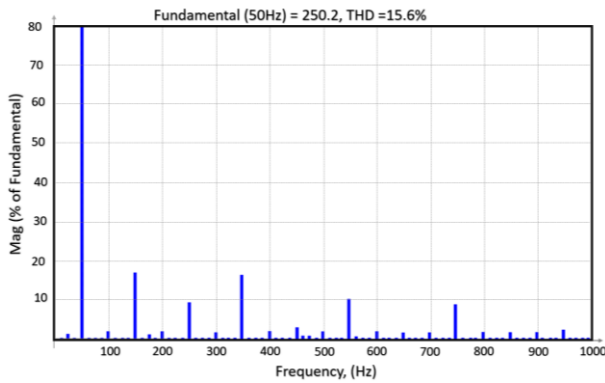


Figure 9. The THD minimization target function FFT Analysis

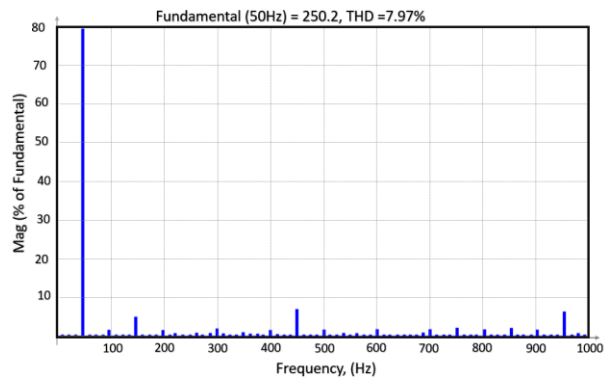


Figure 10. The THD elimination target function FFT Analysis

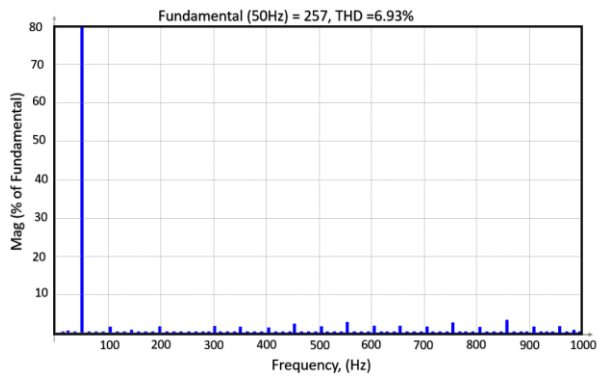


Figure 11. The Optimized Switching Algorithm FFT Analysis

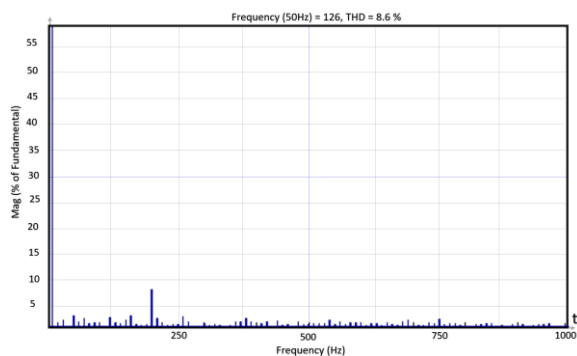


Figure 2. Eleven-level CHBMD output voltage SPWM switching technique FFT analysis

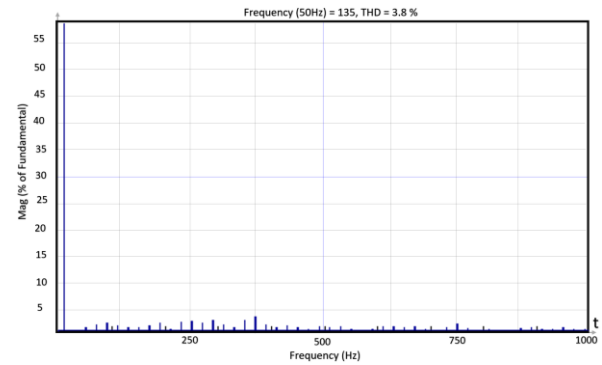


Figure 13. Eleven-level CHBMD output voltage Optimized Switching Algorithm switching technique FFT analysis

The composite objective generated high 3rd- and 5th-order harmonic components of 29.79% and 14.01%, respectively. These dominant low-order components explain the high resulting THD of 33.81%. The proposed mixed-norm method therefore achieved improved aggregate waveform quality by balancing the complete harmonic penalty with fundamental-voltage regulation rather than forcing the exact elimination of only a small number of harmonic orders.

4.5 Performance Over the Modulation-Index Range

The operating range of the proposed mixed-norm switching-angle method was evaluated by varying the modulation index from 0.45 to 1.00. For each modulation-index value, the switching angles were optimized and the resulting output voltage and voltage THD were calculated. The reported results are summarized in Table 6.

Table 6. Output-Voltage and THD Performance Over the Modulation-Index Range

Modulation Index	Output Voltage (V)	Voltage THD (%)
0.45	144.5	13.60
0.50	174.5	12.80
0.55	181.4	10.40
0.60	181.4	10.10
0.65	195.6	8.90
0.70	220.5	7.40
0.75	220.5	6.85
0.80	220.5	6.85
0.85	220.5	6.85
0.90	220.5	6.95
0.95	220.5	6.97
1.00	221.0	6.92

The results identify two principal operating regions. At low modulation indices between 0.45 and 0.65, the voltage THD decreased from 13.60% to 8.90% as the modulation index increased. The higher distortion in this region indicates that the available switching-angle combinations provide a less favorable approximation of the desired sinusoidal waveform. At a modulation index

of 0.70, the THD decreased to 7.40%. Further improvement was obtained between modulation indices of 0.75 and 1.00, where the THD remained within a relatively narrow range of approximately 6.85%–6.97%.

The lowest reported THD in the modulation-index sweep was 6.85%, obtained at $m_a = 0.75, 0.80$, and 0.85. At the upper modulation limit, $m_a = 1.00$, the reported output voltage was 221.0 V and the THD was 6.92%.

The approximately constant reported output voltage between modulation indices of 0.70 and 1.00 indicates that the voltage-regulation component of the objective function maintained the output close to the specified reference.

4.6 Summary of Simulation Findings

The simulation results demonstrate that the mathematical formulation of the objective function has a substantial influence on the optimized switching angles and harmonic performance of the cascaded H-bridge motor drive. Among the five evaluated formulations, the proposed mixed-norm objective achieved the lowest benchmark voltage THD of approximately 6.93%. The proposed method reduced THD by approximately 11.5% relative to the truncated voltage-THD objective and by approximately 13.0% relative to the weighted harmonic-elimination objective. The harmonic-spectrum comparison showed that exact or strong suppression of selected individual harmonics did not necessarily result in the lowest total harmonic distortion. The proposed method provided improved aggregate waveform quality by combining a fourth-order fundamental-voltage regulation term with a quadratic penalty applied to a broader harmonic spectrum. The modulation-index analysis showed that harmonic performance improved as the modulation index increased. The proposed method maintained a comparatively stable voltage THD of approximately 6.85%–6.97% over the modulation-index range of 0.75–1.00 and remained effective over the broader range of approximately 0.70–1.00. Overall, the simulations support the effectiveness of the proposed mixed-norm switching-angle formulation for balancing fundamental-voltage regulation and harmonic reduction under fundamental-frequency switching.

5. EXPERIMENTAL VALIDATION

The proposed mixed-norm switching-angle optimization method was experimentally evaluated using a three-phase eleven-level cascaded H-bridge motor-drive prototype. The experimental study was designed to verify the practical feasibility of the optimized switching angles and to compare the resulting voltage waveform and harmonic performance with those obtained using conventional sinusoidal pulse-width modulation. The laboratory implementation included the CHBMD power stage, isolated dc-link supplies, a digital control unit, switching-signal interface circuits, and a three-phase

motor load. The experimental configuration and the principal numerical values reported in the original manuscript are retained in the following subsections.

5.1 Eleven-Level CHBMD Laboratory Prototype

The experimental platform was based on a three-phase eleven-level CHBMD. Each phase consisted of five series-connected H-bridge power cells. Each individual cell generated three voltage states, $+V_{dc}$, 0, $-V_{dc}$, and the series connection of the five cells produced the eleven-level phase-voltage waveform $-5V_{dc}$, $-4V_{dc}$, $-3V_{dc}$, $-2V_{dc}$, 0, $2V_{dc}$, $3V_{dc}$, $4V_{dc}$, $5V_{dc}$. The modular structure enabled the output waveform to approximate a sinusoidal voltage while limiting the voltage stress across each semiconductor device. The converter was connected to a three-phase motor and operated at a fundamental output frequency of 50 Hz. The laboratory platform included the following principal subsystems:

1. isolated dc-link sources;
2. three-phase cascaded H-bridge power cells;
3. gate-drive and signal-isolation circuits;
4. a digital control circuit;
5. a switching-signal interface;
6. a three-phase motor load;
7. voltage-measurement and waveform-monitoring equipment.

The experimental prototype is shown in Figure 14. The reported operating ratings of the experimental system were 150 W, 380 V, three-phase operation, 50 Hz, 75 r/min, and a power factor of 0.85. These values should be verified against the motor nameplate and laboratory measurements before final submission because the relationship among rated power, rated voltage, and rated speed must be clearly documented.

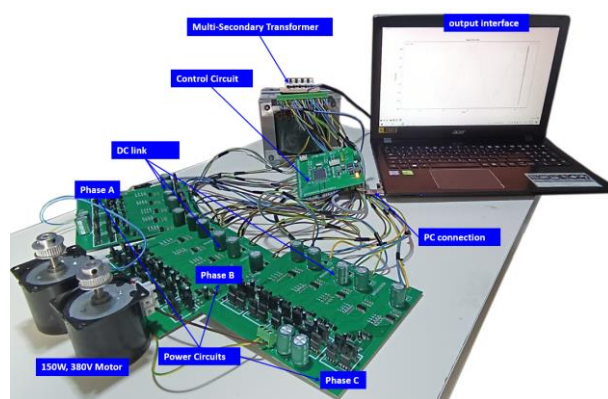


Figure 1. Experimental platform of the three-phase eleven-level cascaded H-bridge motor drive

The final manuscript should additionally identify the following hardware information: power-semiconductor type and part number; dc-link voltage of each H-bridge cell; gate-driver model; controller model; switching-signal voltage level; dead time; voltage and current sensors; oscilloscope or power-analyzer model; motor

rated current and torque. The inclusion of these specifications is necessary to improve experimental reproducibility.

5.2 Controller and Switching-Signal Implementation

The switching signals were generated using the optimized switching angles obtained from the proposed mixed-norm objective function. The numerical optimization was performed in MATLAB, and the resulting switching-angle values were transferred to the MATLAB/Simulink control environment. The switching-angle generation routine was implemented using a MATLAB Function block. The control algorithm determined the switching instants within the first quarter cycle and reproduced the corresponding switching pattern over the complete fundamental cycle using waveform symmetry. For a switching-angle vector $\alpha^* = [\alpha_1^*, \alpha_2^*, \dots, \alpha_s^*]$, the positive-quarter-cycle switching sequence was generated according to $0 < \alpha_1^* < \alpha_2^* < \dots < \alpha_s^* < 90^\circ$.

The switching states were subsequently reflected into the remaining waveform intervals using quarter-wave and half-wave symmetry. For the reported eleven-level operating case, the optimized switching angles were $\alpha_1 = 3.4^\circ$, $\alpha_2 = 13.0^\circ$, and $\alpha_3 = 24.0^\circ$, at a modulation index of $m_a = 0.90$. The current manuscript should explain how these three angles are applied to the five-cell eleven-level converter. An eleven-level CHBMD topology normally provides five switching-angle degrees of freedom when one switching transition is assigned to each cell in the first quarter cycle. Therefore, the final paper must clarify whether:

- three optimized angles are shared among selected cells;
- some cells are switched simultaneously;
- only three effective voltage transitions are used;
- additional angles are fixed;
- a reduced switching pattern is employed.

This explanation is important because the switching pattern must be reproducible.

The switching pulses generated by the digital controller were transmitted to the power stage through the control and interface circuits. Each H-bridge power cell received the corresponding gating signals, producing the required multilevel output-voltage waveform. The motor driver generates the phase output waveform by following the switching pulse pattern, which can be observed on the PC screen as shown in Figure 15 and Figure 16.

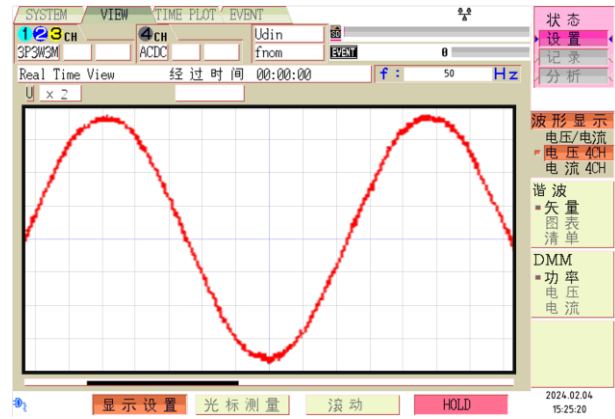


Figure 2. Eleven-level CHBMD output voltage SPWM switching technique test results

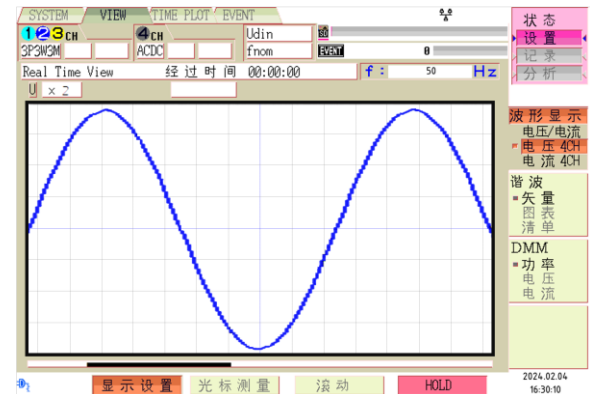


Figure 16. Eleven-level CHBMD output voltage Optimized Switching Algorithm switching technique test results

5.3 Simulation–Experimental Agreement

The simulation and experimental results were compared to evaluate the accuracy of the developed CHBMD model and verify the practical implementation of the optimized switching angles. The simulated model predicted the harmonic behavior of the CHBMD output voltage, while the experimental prototype verified that the optimized switching sequence could be implemented using the available controller and power-electronic hardware. The simulation study reported a minimum THD of approximately 6.93% for the eleven -level model, whereas the eleven-level experimental implementation produced a reported THD of 3.3%. These values should not be directly interpreted as simulation–experimental error because they correspond to different converter levels and operating configurations.

6. DISCUSSION

The proposed mixed-norm method achieved competitive THD with lower online computation than population-based, real-time SHE, and predictive-control approaches, while the eleven-level prototype reduced THD from 8.6% with SPWM to 3.3%. This improvement results from balancing a fourth-order fundamental-voltage error

term with a quadratic harmonic penalty, thereby preserving voltage regulation while suppressing the overall harmonic spectrum. Stable operation was obtained mainly over the modulation-index range of 0.70–1.00, where THD remained approximately 6.85–6.97%. Because the method uses fundamental-frequency switching and offline-optimized angles, it may reduce switching activity and simplify real-time implementation. However, switching losses, computation time, motor-current THD, torque ripple, and high-power performance were not fully evaluated, representing important directions for future work under dynamic industrial operating conditions and unequal dc-link voltages.

7. CONCLUSION

This study addressed the challenge of reducing harmonic distortion in CHBMD while maintaining the required fundamental output voltage and avoiding the excessive switching activity associated with high-frequency modulation methods. To address this problem, a mixed-norm switching-angle optimization approach was developed by combining a fourth-order normalized fundamental-voltage error term with a quadratic harmonic-reduction term. The proposed formulation was designed to provide a balanced compromise between voltage regulation and broad harmonic suppression under fundamental-frequency switching conditions. The proposed objective function was evaluated against four benchmark formulations, including truncated voltage-THD minimization, a composite voltage-regulation and THD objective, selected-harmonic minimization, and weighted harmonic elimination. The simulation results demonstrated that the mathematical structure of the objective function had a direct influence on the optimized switching angles and the resulting harmonic spectrum.

Among the evaluated methods, the proposed mixed-norm formulation achieved the lowest simulated voltage THD of approximately 6.93%, compared with 7.83%, 33.81%, 15.60%, and 7.97% for the benchmark methods. The modulation-index analysis further showed that the proposed method maintained comparatively stable harmonic performance over the range of approximately 0.70–1.00, with THD values remaining close to 6.85–6.97% in the higher modulation region. Experimental validation using an eleven-level CHBMD laboratory prototype confirmed the practical feasibility of the optimized switching-angle strategy. Under the reported operating conditions, the voltage THD was reduced from 8.6% with conventional sinusoidal pulse-width modulation to approximately 3.3% using the proposed method. This result indicates that the proposed formulation can improve output-voltage quality while remaining compatible with low-switching-frequency operation and practical digital implementation. The principal technical contribution of this work is the development of a mixed-norm objective function that jointly considers fundamental-voltage regulation and cumulative harmonic reduction rather than emphasizing only total THD or the elimination of a limited number of harmonic orders. The method may therefore support improved waveform quality in modular CHBMD applications while reducing dependence on high-frequency carrier-based modulation. The present study is limited by the use of different converter configurations for simulation and experimental validation and by the limited evaluation of motor-side variables, switching losses, and computational performance. Future work should therefore focus on matched simulation–experimental validation using the same converter configuration, adaptive weight selection, real-time switching-angle computation, unequal dc-link voltage conditions, and direct assessment of motor-current THD, torque ripple, efficiency, and semiconductor losses.

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