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ADAPTIVE CONTROLLER DESIGN FOR TRAJECTORY TRACKING IN SECOND-ORDER SYSTEMS: APPLICATION TO DC MOTOR VELOCITY CONTROL

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ABSTRACT

This paper presents the design and implementation of an adaptive controller for trajectory tracking in second-order dynamical systems. A theoretical foundation is established by proving that the tracking error dynamics can be reduced to a first-order equation, significantly simplifying the control design. Based on this theoretical framework, a control law is proposed to ensure that the system follows a desired reference trajectory. The proposed controller is implemented on a DC motor to regulate its angular velocity. Experimental results are provided, demonstrating the effectiveness of the controller in achieving precise trajectory tracking under real operating conditions.

1. INTRODUCTION

Control theory has become a cornerstone of engineering and applied sciences, offering rigorous mathematical tools for analyzing and regulating dynamic systems. Its primary goal is to design control laws that ensure stability, robustness, and desired performance, even in the presence of nonlinearities, uncertainties, or external disturbances. Over the past decades, control theory has been applied to a wide range of domains, including industrial machinery, autonomous mobile robots, renewable energy systems, and diverse technological devices. The adaptability of control theory has made it indispensable in both academic research and industrial practice, where the demand for reliable and efficient control strategies continues to grow.

The design and development of theoretical control laws have been extensively studied, with numerous contributions advancing the state of the art. These works have explored nonlinear control, adaptive regulation, and optimization-based approaches, providing frameworks that extend the applicability of control theory to increasingly complex systems. Collectively, they highlight the continuous evolution of the field and its relevance to emerging technological innovations, some noteworthy theoretical control laws can be found in the following studies: (Acosta Lúa et al., 2021; Abban, 2021; Honda et al., 2023; Jimenez-Lizarraga et al., 2015, 2018,

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2019; Shui et al., 2022; Shukla et al., 2024; Singh et al., 2024).

Beyond theoretical advances, implementation and experimental validation are essential for bridging the gap between abstract models and real-world applications. Experimental studies provide evidence of feasibility, reveal practical limitations, and guide refinements in controller design. Several studies have focused on temperature regulation, motor control, and vehicular systems, thereby demonstrating how theoretical concepts can be successfully translated into practical implementations. These contributions underscore the critical role of experimental validation in confirming that control laws are not only mathematically rigorous but also technologically feasible: (Chapa et al., 2023; Nichols et al., 2025; Nizami et al., 2025; Wane et al., 2024; Zekraoui et al., 2025).

This work presents the control of angular velocity in a direct current (DC) motor. The physical setup of the motor used in the experiments is shown in Figure 1, while the corresponding block diagram representing its dynamic behavior is presented in Figure 2. The dynamic of the DC motor system is represented by a second-order differential equation. To simplify the model, order reduction theory was applied, allowing the system to be treated as a first-order one (Kim et al., 2022; Muller et al., 2021; Prajapati & Prasad, 2022; Touzé et al., 2021; Yang et al., 2024).

Building upon this reduced-order model, we propose a novel adaptive controller for trajectory tracking. The controller ensures that the angular velocity of the DC motor follows a desired reference trajectory with high accuracy. Its adaptive nature allows it to adjust to variations in system parameters and external disturbances, thereby guaranteeing robustness. Theoretical analysis and experimental validation confirm that the tracking error converges to zero, demonstrating the effectiveness of the proposed approach. Similar adaptive strategies have been reported in the literature, emphasizing the importance of adaptability in modern control systems, (Montoya-Acevedo et al., 2024; Nizami et al., 2025; Varga et al., 2024).

Figure 1. DC motor

Experimental tests of the adaptive controller implemented on the DC motor demonstrated the functionality and effectiveness of the designed controller under real operating conditions. This research work contributes to the ongoing discourse on control theory by presenting a practical application of adaptive control to DC motor velocity regulation. The integration of order reduction techniques with adaptive trajectory tracking provides a framework that is both theoretically novel and experimentally validated. The results presented herein reinforce the importance of experimental validation and highlight the potential of adaptive control strategies to enhance the performance and reliability of electromechanical systems.

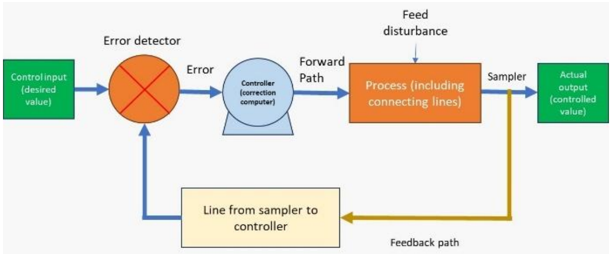

Figure 2. Block diagram of a DC motor system

2. PROBLEM STATEMENT

The control of angular velocity in direct current (DC) motor is a fundamental problem in industrial and robotic applications, where precise tracking of references trajectories is essential. The dynamics of a DC motor can be modeled as a second-order system, where the angular velocity $\omega(t)$ is governed by the following differential equation:

$$\ddot{\omega}(t) + a\dot{\omega}(t) + b\omega(t) = \frac{k_m}{JL}v(t) - \frac{k_m R}{JL}i(t) \qquad (1)$$

Where the parameters represent electrical and mechanical characteristics of the motor, as defined as Table 1.

Table 1. Variables and parameters of the DC motor

Symbol	Description	Units
$\omega(t)$	Angular velocity	rad/sec
$v(t)$	Voltage input	V
$i(t)$	Armature current	A
k_m	Motor torque constant	N. m/A
J	Moment of inertia	$\mathrm{kg \cdot m^2}$
L	Inductance	H
R	Resistance	Ω
B	Friction Coefficient	$\mathrm{N \cdot m \cdot s/rad}$
k_a	Back electromotive force (EMF) constant	$\mathrm{V \cdot s/rad}$

The parameters a and b in equation (1) are system constants, defined as:

$$a = \frac{B}{J}, \qquad b = \frac{k_m k_a}{JL} \qquad (2)$$

Equation (1) was derived by analyzing the electrical circuit and the mechanical dynamics of the motor. A detailed derivation can be found in Appendix A.

To simplify the control design, the input voltage v(t) is proposed as:

$$v(t) = \frac{JL}{k_m}\left(u(t) + \frac{k_m R}{JL} i(t)\right) \qquad (3)$$

where $u(t)$ is a virtual control input. Substituting this into the motor dynamics yields the following simplified second-order system:

$$\ddot{\omega}(t) + a\dot{\omega}(t) + b\omega(t) = u(t) \qquad (4)$$

The control objective is to design $u(t)$ such that the angular velocity $\omega(t)$ follows a reference trajectory $\omega_r(t)$ ensuring that the tracking error $e(t) = \omega(t) - \omega_r(t)$ converges to zero as $t \to \infty$. This is achieved through a trajectory tracking strategy for second-order systems.

3. CONTROL STRATEGY

In this section, we present the theoretical framework for the design of the adaptive controller. The central motivation is to reformulate the system dynamics into a simplified representation that facilitates controller synthesis while preserving the essential characteristics of the original model. Specifically, the dynamics of the DC motor are expressed in terms of a second-order differential equation, which captures both the electrical and mechanical interactions inherent to the system. While such a formulation is accurate, it often complicates the design of control laws due to the presence of multiple coupled variables and higher-order terms.

To overcome these challenges, the key idea is to reduce the system dynamics to a first-order equation. This transformation is not merely a mathematical convenience but a strategic step that enables the application of adaptive control techniques with greater efficiency. By working with a reduced-order model, the complexity of the control design is significantly diminished, allowing for the derivation of analytical results that are easier to interpret and implement in practice.

Moreover, the first-order representation provides a clearer link between the input and output variables, which is particularly advantageous for trajectory tracking problems where the objective is to ensure that the system output follows a desired reference signal. In addition, the reformulation highlights the broader methodological principle of model simplification in control theory. Complex systems often require approximations or reductions to make controller design tractable, and the approach adopted here exemplifies how such techniques can be systematically applied. The reduced-order model not only facilitates theoretical analysis but also enhances the feasibility of real-time implementation, since controllers based on simpler models demand fewer computational resources. This balance between mathematical rigor and practical applicability is a defining feature of modern adaptive control strategies.

The following theorem therefore establishes the fundamental transformation that enables the subsequent development of the adaptive controller. It provides the formal link between the original second-order dynamics and the reduced first-order representation, ensuring that the adaptive control law derived in later sections is both theoretically justified and practically implementable.

3.1 Reduction of Order in Differential Equations

Theorem 1. Consider the second-order differential equation:

$$\ddot{e}(t) + 2c\dot{e}(t) + c^2 e(t) = 0, \qquad (5)$$

where c is constant parameter, $e(0) = e_0$ and $\dot{e}(0) = \dot{e}_0$ are the initial conditions. Then, system (5) is equivalent to the following first-order differential equation:

$$\dot{\sigma}(t) + c\sigma(t) = 0 \qquad (6)$$

where:

$$\sigma(t) = \dot{e}(t) + ce(t), \dot{\sigma}(t) = \ddot{e}(t) + c\dot{e}(t). \qquad (7)$$

Proof: To prove the equivalence, it is necessary to show that systems (5) and (6) are equal. For proving this, we substitute expressions σ and $\dot{\sigma}$ given in (7) into (6) obtaining that:

$$\ddot{e} + c\dot{e} + c(\dot{e} + ce) = \ddot{e} + 2c\dot{e} + c^2 e = 0. \qquad (8)$$

As shown in (8), the equality holds. Hence, the equivalence between systems (5) and (6) has been proven.

Remark 1. Theorem 1 reduces the second-order error dynamics to a first-order equation via $\sigma = \dot{e} + ce$. Unlike sliding mode control, no discontinuous terms appear, avoiding chattering. This is not a robust control approach; it only provides structural conditions for asymptotic stability where the parameter c governs the convergence toward zero (see Section 4.2).

3.2 Solution to the Equivalent First-Order System

Once the system (5) is transformed into its first-order representation (6), the solution for $e(t)$ can be explicitly obtained. The following lemma provides the solution.

Lemma 1. Consider the equivalent first-order differential equation (6) with initial condition $\sigma(0) = \sigma_0$, Then, the solution $e(t)$ to the original system (5) is given by:

$$e(t) = exp(-ct)(e_0(1 + ct) + \dot{e}_0) \quad (9)$$

Moreover, if $c > 0$, the solution $e(t) \to 0$ as $t \to \infty$.

Proof: First, note that (6) is a separable equation, so, by integrating both sides $\int \frac{d\sigma(t)}{\sigma(t)} = -c\int dt + D$, we have $\sigma(t) = D\exp(-ct)$, and applying the initial condition $\sigma(0) = \sigma_0$, we find that $D = \sigma_0$. Thus,

$$\sigma(t) = \sigma_0\, exp(-ct) \quad (10)$$

Being as, $\sigma(t) = \dot{e}(t) + ce(t)$, we have:

$$\dot{e}(t) + ce(t) = \sigma_0\, exp(-ct) \quad (11)$$

Now multiplying both sizes by $exp(ct)$ we have

$$exp(ct) \times \left(\dot{e}(t) + ce(t)\right) = \sigma_0 \quad (12)$$

Using the fact that $\exp(ct) \times (\dot{e}(t) + ce(t)) = \frac{d}{dt}(e(t) \times \exp(ct))$, we have $d(e(t) \times \exp(ct)) = \sigma_0 dt$. Integrating and considering that $e(0) = e_0$, we obtain:

$$e(t) = (\sigma_0 t + e_0)\, exp(-ct), \quad (13)$$

Substituting $\sigma_0 = \dot{e}_0 + ce_0$ into (13) and simplifying, we obtain the expression (9). Finally, taking the limit as $t \to \infty$ and noting that $c > 0$, we observe that $e(t) \to 0$.

Therefore, Lemma 1 has been proven.

3.3 Adaptive Controller Design for Trajectory Tracking

Now, we present the design of the adaptive control law for trajectory tracking. Consider the second-order system:

$$\ddot{x}(t) + a\dot{x}(t) + bx(t) = u(t), \quad (14)$$

where $a, b \in \mathbb{R}$ and with initial conditions $x(0) = x_0$ and $\dot{x}(0) = \dot{x}_0$,

Given a reference trajectory $r(t)$, the control objective is to ensure that $x(t) \to r(t)$ as $t \to \infty$. The following theorem provides a control law that guarantees this objective.

Theorem 2. (Adaptive Controller Design)

If the control input is designed as:

$$u(t) = \ddot{r}(t) + a_m\dot{r}(t) + b_m r(t) + (a - a_m)\dot{x}(t) + (b - b_m)x(t), \quad (15)$$

where a_m and b_m are positive constants selected as $a_m = 2c$ and $b_m = c^2$ with $c > 0$, then $x(t) \to r(t)$ as $t \to \infty$.

Proof: Note, that if we substitute $u(t)$ given in (15) into (14) and we simplify, we obtain:

$$(\ddot{x} - \ddot{r}) + a_m(\dot{x} - \dot{r}) + b_m(x - r) = 0 \quad (16)$$

And defining a new variable e as the error tracking, i.e., $e = x - r$, then $\dot{e} = \dot{x} - \dot{r}$ and $\ddot{e} = \ddot{x} - \ddot{r}$, and given that $a_m = 2c$, $b_m = c^2$ we have:

$$\ddot{e}(t) + 2ce(t) + c^2 e(t) = 0 \quad (17)$$

And according to Lemma 1, if $c > 0$, then $e(t) \to 0$ as $t \to \infty$, which implies that $x(t) \to r(t)$ as $t \to \infty$. Therefore, Theorem 2 has been proven.

Remark 2. The control law $u(t)$ combines feedforward $(\ddot{r} + a_m\dot{r} + b_m r)$ with feedback terms $(a - a_m)\dot{x} + (b - b_m)x$. Adaptation is implicit; no online estimation is required. The design structure $a_m = 2c$ and $b_m = c^2$ with $c > 0$ ensures that the solution $e(t)$ approaches zero asymptotically with a critically damped response, as shown in (13). The selection of c determines the convergence rate toward a neighborhood of zero within a desired time.

4. DESIGN AND IMPLEMENTATION FOR CONTROLLING THE ANGULAR VELOCITY OF A DC MOTOR

To regulate the angular velocity $\omega(t)$ of a DC motor, we consider the second-order system given in equation (4):

$$\ddot{\omega}(t) + a\dot{\omega}(t) + b\omega(t) = u(t)$$

This equation has the same structure as system (14) presented in Section 3.3, which was used to design the adaptive controller given in (15). Based on Theorem 2, we propose the virtual control input $u(t)$ as:

$$u(t) = \ddot{\omega}_r(t) + a_m\dot{\omega}_r(t) + b_m\omega_r(t) + (a - a_m)\dot{\omega}(t) + (b - b_m)\omega(t), \quad (18)$$

where $a_m = 2c$ and $b_m = c^2$. Substituting this control law into (4) we obtain the following closed-loop error dynamics:

$$\ddot{e}_\omega(t) + 2c\dot{e}_\omega(t) + c^2 e_\omega(t) = 0 \quad (19)$$

where $e_\omega(t) = \omega(t) - \omega_r(t)$ represents the angular tracking error. According to Lemma 1, the analytical solution for $e_\omega(t)$ is:

$$e_\omega(t) = exp(-ct)(e_\omega(0)(1+ct) + \dot{e}_\omega(0)) \quad (20)$$

Since $c > 0$, it follows that $e_\omega(t) \to 0$ as $t \to \infty$. This result ensures that the angular velocity of the DC motor $\omega(t)$ converges to the desired angular reference trajectory $\omega_r(t)$.

4.1 Alternative case: Constant angular reference

In the case when the desired angular reference is a constant, i.e., $\omega_r(t) = \omega_r \; \forall t$, it follows that $\dot{\omega}_r(t) = 0$ and $\ddot{\omega}_r(t) = 0$. Substituting these values into the control law (18), we obtain:

$$u(t) = b_m\omega_r + (a - a_m)\dot{\omega}(t) + (b - b_m)\omega(t) \quad (21)$$

Regardless of whether $\omega_r(t)$ is time-varying or constant, the actual voltage input $v(t)$ to the motor must be applied according to equation (3) . It is worth noting that in this work, a constant angular velocity reference was used for the experimental test.

4.2 Selection of c for desired convergence Time

To control the speed of convergence and guarantee that the angular velocity $\omega(t)$ approaches the reference ω_r within a desired finite time T_d, we design the parameter c as:

$$c = -ln(P)/T_d \,, \quad 0 < P < 1 \quad (22)$$

By choosing a small value of P, we ensure that the error $e_\omega(t)$ at time $t = Td$ is sufficiently small. In this work, we propose to set $P = 0.001$.

4.3 Mathematical Justification for Selecting c

The selection of the parameter c plays a crucial role in the performance of the adaptive control law, as it directly governs the convergence rate of the tracking error $e_\omega(t)$.

Let us consider the analytical solution of the error tracking given in (20), and assume the initial conditions:

$$e_\omega(0) = \omega(0) - \omega_r = -\omega_r; \;\; \dot{e}_\omega(0) = \dot{\omega}(0) - \dot{\omega}_r = 0.$$

By substituting these conditions into (20) , and selecting c according to equation (22), we obtain:

$$e_\omega(t) = \exp\left(\frac{\ln(P)}{t_d}t\right)\left(-\omega_r\left(1 - \frac{\ln(P)}{t_d}t\right)\right)$$

Then evaluating the error at $t = T_d$, we obtain:

$$e_\omega(T_d) = P\big(-\omega_r(1 - \ln(P))\big) = P * \gamma,$$

where $\gamma = -\omega_r(1 - \ln(P))$. Since P can be selected arbitrarily small, this ensures that $e_\omega(T_d)$ becomes as close to zero as desired.

Therefore, the selection of c according to equation (22) provides both a mathematical and practical justification for achieving a negligible tracking error within the desired convergence time T_d.

5. EXPERIMENTAL SCENARIO

This section describes the experimental setup and the procedures employed to implement and validate the proposed adaptive controller on a DC motor system.

The description encompasses both the hardware configuration and the software environment used to carry out the tests, ensuring that the methodology is transparent and reproducible. Particular attention is given to the specification of the motor, the sensors, and the data acquisition system, as these elements directly influence the accuracy of the measurements and the reliability of the control implementation.

In addition, the procedures for initializing the system, applying the control signals, and recording the motor's response are outlined step by step, providing a clear framework for replication.

The motor employed in this study is the Traxxas Titan 550. To measure the motor's angular velocity ω, a slotted optical switch (H92B4) was employed in conjunction with a 20-slot encoder disc to convert rotational speed into pulse signals.

The input voltage v_{in} was monitored using a voltmeter connected in parallel with the motor, while the armature current i_a was recorded using an ammeter in series with the power source. Additional current measurements $i(t)$ for controller implementation were obtained using an ACS712 sensor.

These measurements allowed the estimation of the motor's key electrical and mechanical parameters, summarized in Table 2. These values were critical for constructing the mathematical model of the motor and calculating the controller parameters, such as resistance R, inductance L, torque constant k_m, moment of inertia J, and friction coefficient B.

The actual control input voltage $v(t)$ was computed using the virtual controller $u(t)$ defined in equation (21) and translated into a real voltage command through equation (3).

In all experiments, the reference angular velocity was set as a constant:

$$\omega_r = 41.8 \text{ rad/sec} \approx 399.3 \text{ RPM}$$

Table 2. Measured and Calculated Parameters

Parameter	**Sym**	**Value**	**Description**
Resistance	R	4.3Ω	Measured in the motor
Armature current	i_a	0.08A	Measured when the motor starts rotating
Angular velocity	ω	11.52 rad/s	Measured
Input voltage	v_{in}	0.6 V	Measured when ω was taken
Inductance	L	0.0049 H	Measured in the motor
Back EMF constant	k_a	0.02255 $\mathrm{V \cdot s/rad}$	$k_a = \frac{V_{in}-(i_a R)}{\omega}$
Motor torque constant	k_m	0.02255 $\mathrm{N \cdot m/A}$	Assumed equal to k_a
Moment of Inertia	J	0.00015373 $\mathrm{K_g \cdot m^2}$	$J = \frac{t_m \cdot k_a \cdot k_m}{R}$
Torque	τ_m	0.00179 $\mathrm{N \cdot m}$	$\tau_m = k_m \cdot i_a$
Friction coefficient	B	0.00015538 $\mathrm{N \cdot m \cdot s/rad}$	$B = \frac{\tau_m}{\omega}$

Here $t_m = 1.3$ seconds represents the time required for the motor's speed to reach approximately 63% of its final speed following a step voltage input, which was used to estimate the moment of inertia.

6. VALIDATION OF THE ADAPTIVE CONTROLLER: SIMULATION AND EXPERIMENTAL RESULTS

6.1 Simulation Results

Before presenting the experimental results, a preliminary simulation study was conducted to evaluate the theoretical performance of the adaptive controller in tracking the reference angular velocity. This simulation served as an essential step in validating the control strategy under controlled conditions, allowing us to examine its stability, robustness, and convergence properties without the influence of measurement noise or hardware limitations. By analyzing the simulated response, we were able to confirm that the controller achieved accurate trajectory tracking and ensured that the error converged to zero, thereby providing strong evidence of its effectiveness prior to implementation on the physical DC motor system.

Figure 3 shows the simulated response of the system, where the angular velocity $\omega(t)$ successfully converges to the constant reference value $\omega_r = 41.8$ rad/sec, demonstrating the expected exponential decay of the tracking error as established in the theoretical framework.

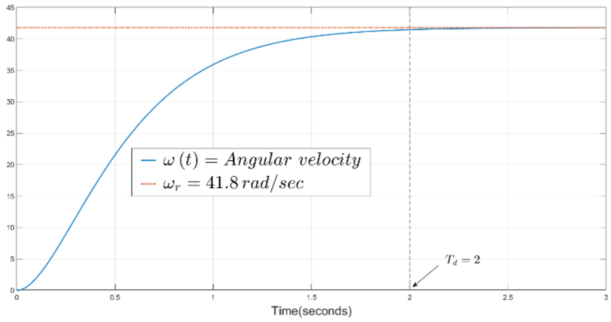

Figure 3. Simulated tracking of the angular velocity reference

6.2 Experimental Results

The reference angular velocity ω_r and all model parameters were incorporated into the adaptive controller, and the performance was evaluated in real-time operation.

To ensure a desired convergence time of $T_d = 2$ seconds, the tuning constant was selected using equation (22), with $P = 0.001$ resulting in $c = 3.4538$. Consequently, the parameters a_m and b_m were calculated as:

$$a_m = 2c = 6.9076, \; b_m = c^2 = 11.9287$$

Using the measured motor parameters listed in Table 2, the constants a and b were calculated via equation (2) as:

$$a = 1.010, \qquad b = 675.05.$$

With these values, the controller was implemented, and its performance was experimentally evaluated.

Figure 4 presents the measured voltage as a function of time. The voltage stabilizes around 2.3 V while maintaining the angular velocity within a narrow band around the reference, between 399 and 414 RPM. These results confirm the controller's capacity to track the constant reference accurately and maintain stability under real operating conditions.

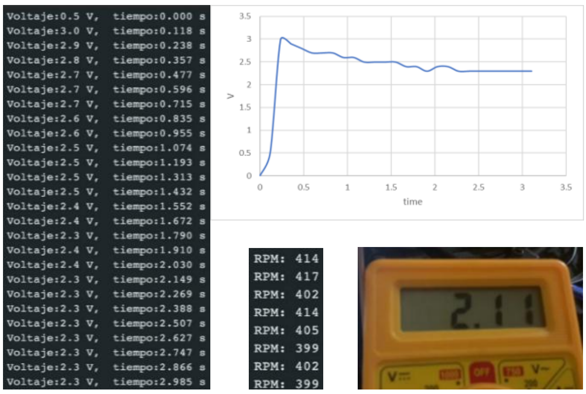

Figure 4. Measured voltage function of time: numerical data and plotted voltage curve

7. ANALYSIS AND DISCUSSION

The experimental results validate the theoretical control strategy presented in earlier sections. As predicted by the analytical solution in equation (20), the tracking error $e_{\omega}(t)$ decays exponentially over time. The controller effectively drives the system's response to match the desired reference, achieving convergence within the expected time T_d.

The voltage in Figure 4 is smooth, confirming that there are no high frequency oscillations, such as the chattering phenomenon. This avoids mechanical wear and electromagnetic interference. Steady-state voltage (~2.3 V) compensates friction implicitly; any mismatch between nominal and true friction is automatically corrected by the feedback terms.

The selection of the parameter c based on the desired precision level P is mathematically justified in section 4.3. The analysis shows that the final error at $t = T_d$ is directly proportional to P, allowing fine control over system precision through simple tuning.

Compared with traditional PID-based methods, the adaptive strategy demonstrated in this study provides a systematic design framework that is explicitly derived from the system's dynamics and offers guaranteed convergence properties. Unlike PID controllers, which rely on heuristic tuning and may struggle to maintain performance under parameter variations or external disturbances, the adaptive approach ensures robustness by continuously adjusting its parameters in response to changes in the system. This feature makes the proposed controller particularly well suited for applications where reliability and precision are critical.

While the current investigation focuses on the tracking of a constant reference signal, the theoretical formulation of the control law is sufficiently general to accommodate time-varying references. This adaptability extends the potential scope of the controller to a wide range of scenarios, including trajectory tracking in robotic systems, speed regulation in electromechanical drives, and performance optimization in renewable energy devices. By demonstrating both systematic design and guaranteed convergence, the proposed adaptive strategy not only addresses the limitations of conventional PID-based methods but also opens avenues for broader applications in control engineering.

8. CONCLUSION

This work presented the design and experimental implementation of an adaptive controller for trajectory tracking in second-order dynamical systems, specifically applied to the angular velocity control of a DC motor. The proposed methodology demonstrated how the reformulation of the tracking error dynamics into an equivalent first-order equation can significantly simplify the control law while retaining rigorous convergence guarantees. This reduction not only facilitated the theoretical analysis but also enabled practical implementation, ensuring that the controller could be deployed in real-time applications without excessive computational burden.

The proposed controller was implemented using a constant reference signal, and the experimental results validated its effectiveness in achieving accurate trajectory tracking under real-world conditions. The systematic selection of the tuning parameter c, based on the desired convergence time T_d, proved to be a practical and mathematically justified approach to ensure rapid and precise error reduction.

Overall, the proposed methodology offers a robust and adaptable solution for controlling second-order systems, with potential applications in various domains such as robotics, industrial automation, and mechatronic systems. Future work may extend this approach to time-varying reference signals, nonlinear systems, or multi-variable control scenarios.

A particularly noteworthy aspect is the clear connection between c and the experimentally observed convergence rate: the controller guarantees percentage convergence P within a desired time T_d, as formalized in Section 4.3. This structured methodology eliminates trial-and-error tuning, unlike conventional PID methods that require multiple iterations to balance rise time, overshoot, and steady-state error. Furthermore, stability is guaranteed for any $c > 0$, making the tuning process both safe and predictable.

From an educational perspective, the order-reduction strategy presented in Theorem 1 offers a valuable pedagogical tool for teaching advanced control concepts. Students can appreciate how a linear transformation simplifies a second-order system into a first-order one without resorting to state-space methods or Lyapunov theory. The experimental validation on a low-cost DC motor platform further reinforces the connection between theory and practice, making this approach accessible to undergraduate and graduate laboratories.

In summary, this work contributes not only a novel adaptive controller but also a systematic design methodology that balances mathematical rigor with practical feasibility. The successful experimental results encourage further exploration of order-reduction techniques in other classes of dynamical systems, including time-delay systems, fractional-order systems, and nonlinear mechanical systems.

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Appendix A. Derivation of the Angular Velocity Model

The mathematical model of the DC motor is obtained from its electrical and mechanical dynamics, given by:

$$L\frac{di(t)}{dt} = v(t) - Ri(t) - k_a\omega(t) \tag{23}$$

$$J\frac{d\omega(t)}{dt} = k_m i(t) - B\omega(t) \tag{24}$$

Differentiating equation (24) with respect to time:

$$J\frac{d^2\omega(t)}{dt^2} = k_m\frac{di(t)}{dt} - B\frac{d\omega(t)}{dt} \tag{25}$$

Substituting $\frac{di(t)}{dt}$ from equation (23) into (25) and simplifying we obtain the equation (1) that describes the angular velocity dynamics of the motor.