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FREE VIBRATION OF STIFFENED P-FGM PLATES

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A B S T R A C T

The vibration behavior of stiffened plates is crucial for engineering structures like ships, aircraft, platforms, and buildings, directly impacting integrity, noise control, and design optimization. This research comprehensively investigates the free vibration characteristics of stiffened Functionally Graded Material (FGM) plates, where the volume fraction of Mo (bottom) and AlN (top) varies through the thickness following a power-law (P-FGM). A Mo stiffener is placed at the center line on the metallic side. Numerical analysis employs the finite element method (FEM), with an optimization study of material property averaging methods - rule of mixture, Mori-Tanaka, Reuss, LRVE, and Hashin-Shtrikman bounds. The Mori-Tanaka model proves most accurate for graded layers and is used for simulations. Model validity is ensured through experiments on stiffened steel plates for selecting appropriate finite element types. The validated approach is applied to study how volume fraction gradient, plate geometry, and stiffener geometry affect vibration behavior in stiffened P-FGM plates.

1. INTRODUCTION

These The study of vibration behavior of stiffened plates is crucial for engineering applications due to the roles these structures play in various industries such as shipbuilding, aerospace, and civil engineering. The necessity arises from several intertwined factors. Stiffened plates are prone to large vibration amplitudes because of their low inherent damping. Excessive vibration not only leads to structural fatigue and failure but also increases noise, which can negatively affect comfort and performance requirements - especially in ships, aircraft, and offshore platforms. Understanding vibration behavior enables effective strategies for noise and vibration reduction, enhancing the environment and functionality of the structure (Zhang et al., 2024; Sun, 2022; Hamedani et al., 2012). Analyzing vibration characteristics helps prevent resonance and dynamic instability, which are dangerous failure modes in load-bearing structures. Proper analysis ensures the structure can carry dynamic loads safely, maintaining stability, load-bearing capacity, and seismic resistance (Hamedani et al., 2012). The performance of stiffened plates depends on parameters such as location, number, dimensions, and orientation of stiffeners. Vibration studies allow engineers to optimize these parameters for improved structural efficiency and material economy, without compromising safety or performance (Zhang et al., 2024; Hamedani et al., 2012). In efforts to reduce structural weight (e.g., in shipbuilding or aerospace), the use of composite materials and optimized stiffener configurations is common. However, lightweight structures can also become more vibration-sensitive, making accurate vibration analysis critical to ensure their practical viability. Knowing how stiffeners affect modal parameters helps in selecting cost-effective vibration

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mitigation solutions - whether passive (adding mass/stiffness) or active (controlled force application) (Hamedani et al., 2012). In summary, a thorough study of vibration behavior in stiffened plates is essential for ensuring safety, reliability, comfort, regulatory compliance, and cost-effective design across a wide spectrum of engineering applications (Zhang et al., 2024; Hamedani et al., 2012).

Functionally Graded Materials (FGMs) are advanced engineering composites made from two or more phases with continuously varying composition. First introduced in the early 1980s during a Japanese space plane project, FGMs were developed to withstand extreme temperature variations. These materials feature gradients in composition, structure, or properties along a preferred direction, making them superior to homogeneous materials. Mechanical properties such as Young's modulus, Poisson's ratio, and shear modulus vary gradually in FGMs, enhancing structural performance. Researchers are customizing FGM architectures at the microscopic level to optimize functional properties for specific applications. FGMs are now widely used across engineering and technology fields due to their ability to tailor material properties. Their development stems from ongoing research in material mechanics, supporting applications in spacecraft, thermal barrier coatings, nuclear components, and more (Mahamood & Akinlabi, 2017). FGMs offer several advantages, including reduced in-plane and transverse stresses, improved thermal properties, and high toughness. Composed of metallic and ceramic phases, they enable smooth transitions in material behavior, reducing issues like cracking and delamination found in traditional layered systems, especially in thermal-barrier applications (Jha et al., 2013; Patel, 2025).

Rikards et al. (2001) presented the development of a triangular finite element for analyzing buckling and vibrations of laminated composite stiffened shells using first-order shear deformation theory (FSDT). The study compared three approaches for laminated structures: single layer, equivalent layer, and layer wise theories. Numerical examples demonstrated the element's accuracy and convergence. The findings underscore the advantages of using refined shell theories for aerospace applications, where laminated composites offer superior structural efficiency. Peng et al. (2006) introduced a mesh-free Galerkin method for analyzing the buckling and free vibration of eccentrically stiffened plates using FSDT. The model allows for flexible placement of stiffeners without mesh constraints, enhancing computational efficiency. Numerical examples demonstrated accuracy, with results aligning closely with existing solutions and ANSYS data. The free vibration characteristics of stiffened plates using finite element analysis, focusing on fundamental frequency and mode shapes are investigated by Nayak et al. (2018). Key parameters include stiffener type, orientation, number, boundary conditions, and the depth-to-thickness ratio. Results indicated that the fundamental frequency increases with the number of stiffeners, particularly with eccentric configurations, outperforming concentric ones. Design charts are proposed for quick frequency determination, aiding designers in practical applications.

Pietrzak et al. (2007) investigated the role of an Al_2O_3-Cr functionally graded interlayer in mitigating thermal residual stresses in Al_2O_3-heat resisting steel joints. Using FEM simulations validated with experimental results, they showed that varying the profile and thickness of the FGM interlayer significantly reduces the residual stresses at the ceramic-metal interface. The study emphasized that tailoring the internal layer profile through the gradient exponent can optimize stress distribution, thereby shifting stress concentration away from weaker zones into stronger intermediate layers. Chi and Chung (2006a, 2006b) conducted a thorough analytical and numerical analysis on FGM plates subjected to transverse loads. In Part I, they derived closed-form solutions for FGM plates using power-law, exponential, and sigmoid volume fraction profiles. The authors demonstrated that while all three profiles affect the bending stiffness differently, the sigmoid function offers a smoother stress transition across layers. Part II extended the analysis with numerical simulations using MARC FEM software. Their results confirmed the analytical solutions, revealing that S-FGM plates offer improved stress and displacement distributions compared to P-FGM and E-FGM types under uniform loads. Building upon these foundations, Talha & Singh (2010) employed higher-order shear deformation theory integrated with FEM to analyze the static and dynamic response of FGM plates. The authors proposed a modified displacement field and a 13-degree-of-freedom per node finite element to capture transverse shear deformation effects accurately. Their results highlighted that the natural frequency of FGM plates increases with a lower volume fraction index and smaller thickness ratios, while static deflection increases with the volume fraction index. This study reinforced the importance of gradient design and higher-order modeling in accurately capturing FGM behavior, especially in thick plates.

Yeilaghi and Kapania (2012) and Gayathri and Parvathy (2023) confirm the functional dependence of structural behavior on material gradation, though their focuses differ: Yeilaghi and Kapania (2012) explore dynamic response using mesh-free EFG modeling, while Gayathri & Parvathy, 2023 emphasize stability under compressive loading via FEM. Notably, both works underscore the nonlinear effects of stiffener geometry and material gradation profiles on FGM structural performance, providing critical insights for the optimal design of advanced aerospace and mechanical components.

The present study investigates the free vibration behavior of stiffened P-FGM plates using finite element analysis and identify the influence of material gradation, plate geometry and stiffener geometry on their dynamic response.

Nomenclature

E	Young's modulus
ϑ	Poisson's ratio
ρ	Mass density
p	Volume fraction index
a	Length of main plate
b	Width of main plate
h	Thickness of main plate
h_s	Height of stiffener
b_s	Width of stiffener

2. OPTIMIZATION STUDY OF MATERIAL PROPERTY AVERAGING METHOD FOR FGMS

Most functionally graded materials consist of two phases: ceramic and metal. The characteristics of FGMs gradually change along one or more directions. For modelling this graded material, analytical approaches such as finite element method and micromechanical models are commonly employed. The assessment of effective material properties, such as E, ϑ, ρ, etc., in these graded composites is conducted by relying on the volume fraction distribution of the constituents. Thus, it is crucial to ascertain the volume fraction distribution of phase constituents within functionally graded materials. Therefore, utilizing mathematical models for graded materials with parallel homogeneous layers and specific material properties is beneficial. Appropriate averaging techniques are used for assessing the characteristics of each layer. Some of the volume fraction distribution functions and effective property averaging methods (Gayathri & Beena, 2024) are:

- Power law (P-FGM)
- Sigmoid law (S-FGM)
- Exponential law (E-FGM)
- Rule of mixture (ROM)
- Mori-Tanaka model
- Reuss model
- Cubic local representative elements model (LRVE)
- Hashin-Shtrikman bounds model

Chen et al. (2020) fabricated AlN/Mo FGMs by spark plasma sintering. The distribution of volume fraction was in accordance with power law model. Four kinds of FGMs with different material gradations were considered. The thickness of each layer was computed by following the procedure explained by Grujicic and Zhao (1998). Accordingly, the thickness of each layer was calculated for different p values. FGM specimens with dimension 20x4x3 mm (length; width; thickness) was considered for three-point bending test as done by Chen et al., 2020. The FE model of the specimens was generated using the software package ABAQUS. FGMs were modelled using S4R element from ABAQUS element library. The effective material properties like E and ϑ were calculated using the different averaging methods and were applied to the FE models of AlN/Mo FGMs. The percentage difference between the present value and experimental value are as given in Table 1.

Table 1. Percentage difference in bending strength

Averaging Method	p<1(0.15)	p=1	p>1(2.07)	p=1*
ROM	24.175	22.919	23.700	23.113
Mori-Tanaka	20.597	19.734	20.322	19.923
Reuss model	24.236	22.989	23.752	23.182
LRVE	24.175	22.919	23.700	23.113
Hashin-Shtrikman lower bound	20.597	19.734	20.322	19.923
Hashin-Shtrikman upper bound	20.597	19.734	20.322	19.923

From Table 1, it is clear that the Mori-Tanaka and Hashin-Shtrikman model gives similar values. Also, they give the bending strength with minimum error to the experimental value for all the different material gradients. For p=1*, the error is a bit more compared to p=1. That is, as the number of graded layers increased, these functions became more efficient in exhibiting the actual condition of material gradation. All these findings can be attributed to the arithmetic mean E in a FGM which is obtained from each method. The average E given to the FGM structure by Mori-Tanaka model and Hashin-Shtrikman model are minimum. The bending strength increases with E. Hence, the both with minimum E gives the lowest bending strengths, which lied close to the experimental value, so the error is minimum. Thus, these two averaging methods exhibited the behavior closer to the real scenario compared to the other methods. Due to the complexity of the equation in Hashin-Shtrikman model, Mori-Tanaka model will be used for further study.

3. VALIDATION OF NUMERICAL MODEL

To validate the finite element model of the stiffened FGM plate under free vibration conditions, an experimental investigation was conducted on stiffened steel plates. Subsequently, the experimental results were compared with finite element analyses employing different element types for both the main plate and the stiffener. This comparative study ensures that the modelling approach accurately captures the interaction between the stiffener and the main plate, thereby confirming the model's capability to replicate the actual dynamic behaviour of the stiffened plate system.

4.1 Material testing

The mechanical properties of steel were assessed through a tensile test, conducted as per the standard tensile test procedure as in ASTM E8 (ASTM International. (2013).), using a computer-controlled UTM of capacity of 100kN. The experimental setup and the eventual failure of the specimen are given in Figure 1 (a) and (b), respectively.

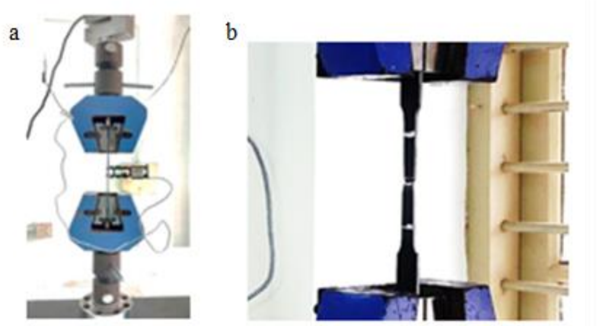

Figure 1. (a) Test setup for tension test; (b) Tension test specimen after testing

The engineering stress-strain curve and the material properties obtained from the tensile test are given in Figure 2 and Table 2, respectively.

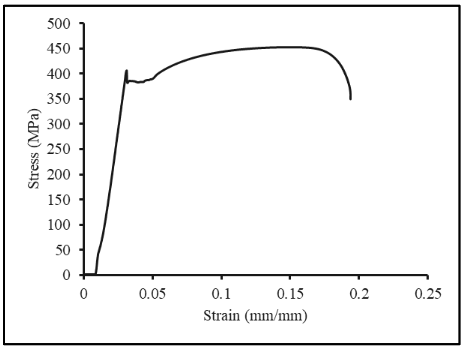

Figure 2. Engineering stress-strain curve obtained for the steel material from the tension test

Table 2. Material properties obtained for steel material from the tension test

Property	Value
Young's Modulus (N/mm^2)	1.7x10^5
Yield Strength (N/mm^2)	384.693
Ultimate Strength (N/mm^2)	453.467

Since the geometry of the specimens are changed gradually from their original value, the engineering stress-engineering strain curve is interfered beyond the elastic stage. The plastic strain, true stress and true strain based on the actual dimensions of the specimen is required in the FE software ABAQUS for conducting numerical simulations.

4.2 Vibration test

Free vibration of stiffened steel plate with thickness 3mm was carried out to find the natural frequency. A stiffener plate is attached to the main plate along its central line. The geometry dimensions and the experimental setup were as displaced in Figure 3 and Figure 4, respectively.

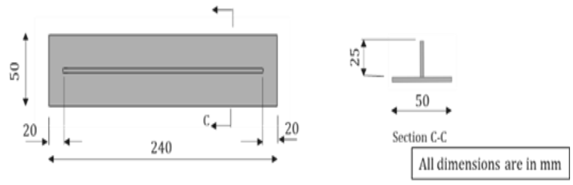

Figure 3. Geometry dimensions of stiffened steel plate specimen for free vibration

Figure 4. Test setup for free vibration

The specimen was clamped with 20mm from each plate end, leaving the other edges free. An accelerometer, adhered at the geometric centre, connects to a signal-conditioning unit (NI DAQ 4431). The impact hammer strokes a selected point five times, averaging the frequency response function input to the computer. Precautions ensured the hammer strikes the plate surface perpendicularly. MEscope software recorded and processed the signals to extract frequencies.

4.3 Finite element modelling of steel plates

The basis of FEM is the representation of a structure by an assemblage of subdivision called finite elements. For stiffened plates, selecting the appropriate element type for the main plate and stiffener is crucial. Various idealizations allow assemblies to be treated as shell and beam elements. Depending on their dimensions, stiffeners can be modelled as either shell or beam elements.

Stiffened plates considered for experimental study were modelled in ABAQUS using different element assemblies as [main plate,stiffener] – [S4R,S4R], [S8R,S8R], [S8R5,S8R5], [S4R,B31], [S8R,B32], [S8R5,B32]. The Lanczos method is used for frequency extraction.

Natural frequency obtained for stiffened plates using different element assemblies in ABAQUS and from experiment are given in Figure 5. Form the chart it is clear that FE model of stiffened plate meshed with S8R5 element for plate and stiffener resulted in frequency close to the experiment value of 917Hz. Table 3 shows the comparison of natural frequencies obtained for the present FE model (with [S8R5,S8R5] element assembly) and other researches. The table clearly shows that the frequencies are in good agreement.

Table 3. Natural frequency obtained for stiffened steel plates form ABAQUS with different element assemblies and experiment

Element combination [main plate,stiffener]	Natural frequency (Hz) from FEM	Natural frequency (Hz) from experiment	% Error to expt.
[S4R,S4R]	933.49	917	1.80
[S8R,S8R]	927.23		1.12
[S8R5,S8R5]	**925.80**		**0.96**
[S4R,B31]	1060.10		15.61
[S8R,B32]	1516.40		65.37
[S8R5,B32]	949.69		3.56

Table 4. Natural frequencies for stiffened steel plates form ABAQUS with [S8R5,S8R5] element assembly and other researches

Mode	Frequency (Hz)				
	Rikards et al., 2001	Peng et al., 2006	Nayak et al., 2018	Olson & Hazell, 1977	Present FEM
1	722.8	574.11	728.8	689	716.82
2	754.6	754.35	749.4	725	746.33
3	997.9	846.55	993	961	987.87
4	1009	993.47	1000	986	998.37
5	1425	1293.8	1409.1	1376	1405.4
6	1430	1402.8	1411.7	1413	1409.3
7	1690	1650.3	1634.7	1512	1604.2
8	1866	1805.1	1893.7	1770	1843.7
9	2027	1897.2	1997.8	1995	1993.7
10	2029	1904.4	1999	2069	1995.8

4. PARAMETRIC STUDY

The optimized material property averaging method for FGM formulation and the validated modelling approach was then applied to analyse the influence of key parameters – volume fraction index, plate aspect ratio, stiffener height, stiffener width and plate thickness on the free vibration behaviour of stiffened P-FGM plates.

4.1 Effect of volume fraction index (*p*)

Square P-FGM plate of *a/b*=1 with a gradual material gradation from AlN at top to Mo at bottom. A stiffener of Mo was attached to the bottom of the plate along the centre line. The equivalent material property like *E* and ϑ was calculated using the Mori-Tanaka model. Both the main plate and stiffener was modelled using S8R5 element. All the four plate edges were fixed. The parameters like *a/b*, plate thickness and stiffness geometry remain constant and *p* is varied from 0 to 8. The results obtained are shown in Figure 6.

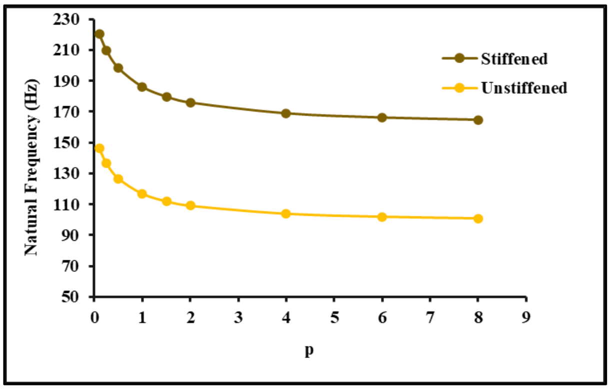

Figure 6. Natural frequencies for stiffened and unstiffened P-FGM plates (*a/b*=1; *b/h*=100; h_s/h=5; b_s/h=2)

As the volume fraction index increases, the natural frequency decreases for both stiffened and unstiffened plates. The decrease is initially steep at lower values of *p* (from 0 to about 2), then becomes more gradual as *p* increases further. Stiffened plates consistently exhibit higher natural frequencies compared to unstiffened plates across all *p* values. The gap between the natural frequencies of stiffened and unstiffened plates is significant and remains fairly consistent as *p* increases. Both curves (stiffened and unstiffened) show a rapid decrease in natural frequency at small values of *p*, which flattens out as *p* increases. This suggests the plate's dynamic response becomes less sensitive to further increases in *p* beyond a certain threshold. Stiffening the P-FGM plate enhances its structural rigidity, leading to a higher natural frequency and thus greater resistance to dynamic excitation. The volume fraction index typically represents the gradation in material composition; increasing it alters the material distribution, often resulting in a reduction of global stiffness, hence the decline in natural frequency. The initial sharp drop indicates that the plate properties are most affected by *p* in the lower range, while at higher *p* values, plate behavior stabilizes.

4.2 Effect of plate aspect ratio (*a/b*)

Here P-FGM plates with *p*=0.1,1,4 was considered. The plate aspect ratio was varied from 0.25 to 4. Other parameters were kept constant. The results obtained are shown in Figure 7 and Figure 8.

The natural frequency of stiffened FGM plates decreases sharply as the aspect ratio increases. For *p*=0.1, the natural frequency is highest for each *a/b* value. As *p* increases to 1 and then 4, the natural frequency consistently decreases for corresponding aspect ratios. Frequencies are very high, especially for *p*=0.1, often exceeding 1000 Hz. The frequencies converge and the difference between p values diminishes, all dropping below 200 Hz for *a/b*=4. Plates with smaller *a/b* ratios (shorter) are much stiffer, leading to much higher natural frequencies, especially when the material gradation parameter p is lower.

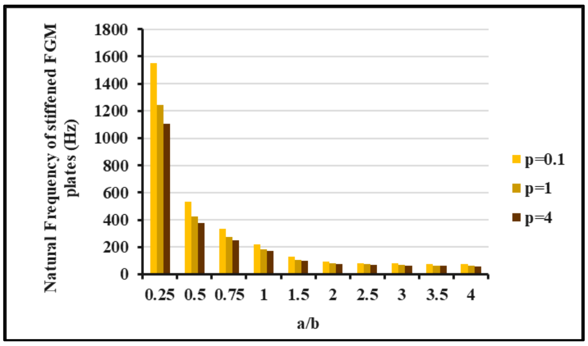

Figure 7. Natural frequencies for stiffened P-FGM plates (p=0.1,1,4; b/h=100; h_s/h=5; b_s/h=2)

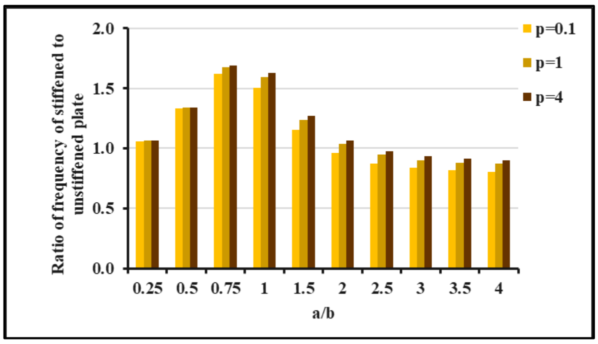

Figure 8. Ratio of natural frequency of stiffened to unstiffened P-FGM plates (p=0.1,1,4; b/h=100; h_s/h=5; b_s/h=2)

The frequency ratio is above 1.0 up to a/b=2, indicating that stiffening raises the natural frequency compared to unstiffened plates. Maximum ratios are observed around a/b values of 0.75 to 1.25, with all p values showing a similar qualitative trend. At all aspect ratios, p=4 provides the highest frequency ratio, followed by p=1 and then p=0.1. At very low (a/b=0.25) and very high ($a/b\geq3$) aspect ratios, the benefit of stiffening (frequency ratio) diminishes and the ratios level out near 1.0.

4.3 Effect of stiffener height (h_s/h)

Here P-FGM plates with a/b=0.25,1,4 was considered. The stiffener height to plate thickness ratio (h_s/h) was varied from 0 to 18. Other parameters were kept constant. The results obtained are shown in Figure 9.

The graph displays the dependence of frequency on the ratio of stiffener height (h_s) to plate thickness (h). For a/b=0.25, frequency initially drops with the stiffener addition and then increases with stiffener height up to a certain point, after which it decreases - identifying an optimal stiffener height. For a/b=1, the frequency broadly increases and then stabilizes or falls at high values of h_s/h. But for a/b=4, no effect with any stiffener height. A moderate increase in stiffener height boosts stiffness and frequency, but excessive height can introduce local flexibility or excessive added mass, causing the frequency to drop.

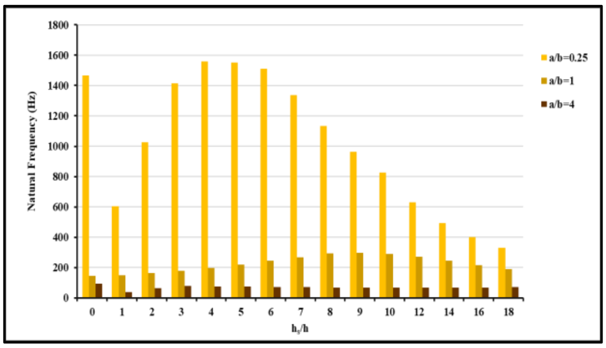

Figure 9. Natural frequencies for stiffened P-FGM plates (p=0.1; a/b=0.25,1 4; b/h=100; b_s/h=2)

4.4 Effect of stiffener width (b_s/h)

Here P-FGM plates with a/b=0.25, 1,4 were considered. The stiffener width to plate thickness ratio (b_s/h) was varied from 0 to 5. Other parameters were kept constant. The results obtained are shown in Figure 10.

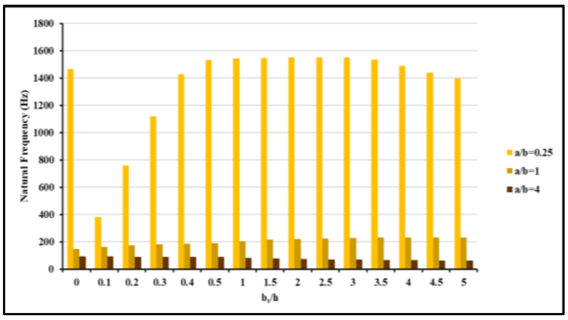

Figure 10. Natural frequencies for stiffened P-FGM plates (p=0.1; a/b=0.25,1 4; b/h=100; h_s/h=5)

For a/b=0.25, frequency increases sharply as stiffener width increases from 0 to 0.5, then levels off - showing that initial stiffening gives significant benefits but the effect plateaus. For a/b=1, frequency gains are modest but present, with no sharp peaks. And for a/b=4, there is no effect of stiffener addition. Adding width to stiffeners greatly increases plate stiffness (and thus frequency) for compact plates. Beyond a certain width, gains diminish due to the mass increase outweighing stiffness benefits.

4.5 Effect of plate thickness (b/h)

Here P-FGM plates with a/b=0.25, 1,4 were considered. The plate width to thickness ratio (b/h) was varied from 12.5 to 400. Other parameters were kept constant. The results obtained are shown in Figure 11 and Figure 12.

At a/b=0.25, plates exhibit the highest natural frequencies, with values exceeding 1400 Hz for smaller b/h and gradually decreasing as b/h increases. For a/b = 1 and a/b = 4 show consistently lower frequencies, emphasizing the strong influence of plate geometry. All cases show a declining frequency as b/h increases, indicating that making the plate thinner decreases the natural frequency. Short, wide plates (low a/b) are inherently stiffer, resulting in higher natural frequencies. As plates become narrower relative to their thickness, their ability to resist bending decreases, leading to lower frequencies.

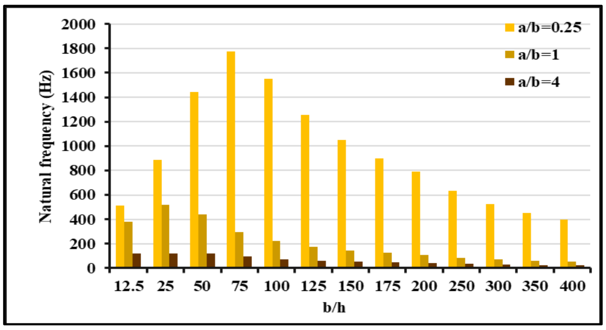

Figure 11. Natural frequencies for stiffened P-FGM plates (p=0.1; a/b=0.25,1 4; h_s/h=5; b_s/h=2)

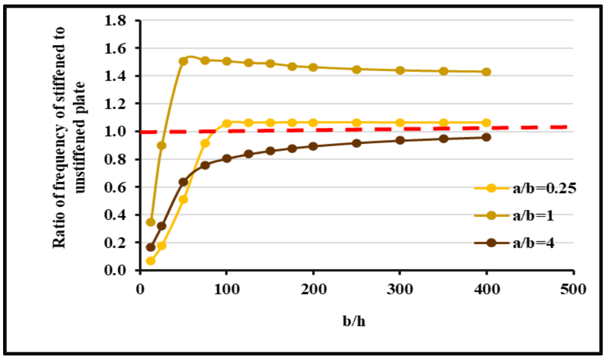

Figure 12. Ratio of natural frequency of stiffened to unstiffened P-FGM plates (p=0.1; a/b=0.25,1 4; h_s/h=5; b_s/h=2)

The vertical axis in Fig. 12 represents the ratio of the natural frequency of stiffened plates to that of unstiffened plates. For a/b=0.25 (short, wide plate), the frequency ratio rises rapidly, peaking at about 1.5 for b/h > 50, and stays nearly constant with further increases. For a/b=1, the ratio increases quickly at low b/h, stabilizing around 1.1 at higher b/h.

For a/b=4 (long, narrow plate), the ratio increases more gradually, approaching but not exceeding 0.9. Stiffening is most beneficial for plates with low aspect ratios (short and wide). The effect saturates for thick plates (high b/h), indicating diminishing returns from further increasing plate thickness for stiffening effectiveness.

5. CONCLUSIONS

The free vibration behavior of stiffened P-FGM plates using finite element analysis was studied and identified the influence of material gradation, plate geometry and stiffener geometry on their dynamic response. The following key conclusions have been drawn:

- Among the various homogenization techniques to evaluate the equivalent material property of each graded layer of FGM investigated, the Mori-Tanaka method provided the most accurate prediction by comparing to the experiment done by Chen et al. (2020).
- The finite element model with the main plate and stiffener modeled with the element S8R5, validated through experimental testing of stiffened steel plates, proved reliable for the vibration analysis of stiffened structures.
- Increasing the volume fraction index (p) in P-FGM plates results in a pronounced reduction in natural frequency for both stiffened and unstiffened configurations. The effect is most notable at lower p values (0 - 2), where the decrease is steep, and it becomes more gradual for higher p. Stiffened plates consistently demonstrate higher natural frequencies than unstiffened ones, with the advantage of stiffening remaining fairly constant as p increases. The initial sensitivity to p indicates significant influence of material gradation on structural stiffness at low indices, while beyond a certain p, further changes minimally impact dynamic behavior. This result supports previous findings on the strong role of material distribution in vibration characteristics of FGM structures.
- The plate aspect ratio (a/b) substantially governs the free vibration response, with natural frequencies sharply decreasing as a/b increases. Plates with smaller a/b ratios (shorter and wider) are much stiffer and possess higher frequencies, particularly at low p values. As a/b increases, frequencies for different p values converge. The benefit of stiffening, measured by the frequency ratio, is most significant at moderate aspect ratios (a/b ~ 0.75–1.25), and diminishes at extreme low or high aspect ratios, reflecting the interplay between geometry and stiffening effectiveness.
- The stiffener height (h_s/h) exhibits a non-monotonic effect on natural frequencies. For short, wide plates (a/b=0.25), there is an optimal stiffener height where the increase in stiffness is maximized before mass addition or local flexibility dominates, reducing frequency. For square plates (a/b=1), frequency increases with stiffener height to a plateau, with diminishing returns at high h_s/h. For long, narrow plates (a/b=4), stiffener height introduces negligible changes. These patterns highlight the necessity of optimizing stiffener dimensions based on the plate's geometric proportions.
- Stiffener width (b_s/h) shows significant frequency gains upon initial increase, especially for compact, wide plates, with the effect saturating at higher widths. In square and elongated plates, the impact is less pronounced,

confirming that both the size and placement of stiffening must be tailored to the plate geometry to avoid unnecessary mass increases that can negate stiffness-related gains.

- Plate thickness (*b/h*) inversely influences natural frequency: as plates become thinner (higher *b/h*), natural frequencies decrease. This effect is most prominent in short, wide plates, where stiffening remains most effective, especially for thick plates. For long, narrow plates, the benefit of stiffening is limited, and the impact on frequency ratio is marginal. Thus, plate geometry and thickness ratio are crucial in determining the overall efficacy of stiffening in vibration control.
- In summary, the free vibration behavior of stiffened P-FGM plates is strongly influenced by material gradation (*p*), geometric proportions (*a/b*, *b/h*), and stiffener parameters (h_s/h, b_s/h). Stiffening is most beneficial for plates with low aspect ratios and moderate thickness, and there exists an optimal stiffener size beyond which benefits diminish. These findings provide a foundation for optimal design and material selection in engineering applications involving P-FGM structural elements and align with trends reported in recent studies on functionally graded material plates.

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