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CONSISTENCY IMPROVEMENT METHOD FOR FUZZY PAIR-WISE COMPARISON MATRIX IN ANALYTIC HIERARCHY PROCESS

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A B S T R A C T

Maintaining consistency is a crucial element in ensuring the reliability of pairwise comparisons provided by experts in the Analytic Hierarchy Process (AHP) and Fuzzy AHP. Many researchers have put forth various definitions concerning the consistency of fuzzy Pairwise Comparison Matrices (PCMs) using fuzzy set theory. However, in most applications of fuzzy AHP, fuzzy PCM consistency is evaluated by defuzzifying the fuzzy comparisons, similar to the approach used for crisp PCMs. This paper introduces a novel method for transforming fuzzy comparisons into crisp comparisons through defuzzification, specifically utilizing the geometric mean. Furthermore, it proposes a method to enhance the consistency of the PCM. The presented methodology is applied to several problems previously addressed in the literature.

1. INTRODUCTION

The Analytic Hierarchy Process (AHP), as introduced by Saaty in 1980, represents a sophisticated approach to Multi Criteria Decision Making (MCDM), encompassing both qualitative and quantitative factors. In the AHP framework, decision makers are tasked with providing judgments regarding paired comparisons of objectives, forming a matrix known as the Pair-wise Comparison Matrix (PCM). In complex systems, decision maker judgments often exhibit linguistic or vague patterns. Consequently, several methods have been devised to address such situations, including AHP with interval judgments, Fuzzy AHP, and Hesitant AHP. Saaty and Vargas (1987) pioneered the incorporation of interval judgments into AHP and derived interval weights through the Monte Carlo simulation method. Many subsequent researchers have also explored interval AHP (e.g., Salo and Hamalainen in 1992, Islam et al. in 1997, Wang et al. in 2005). In 1983, Van Laarhoven and Pedrycz were the first to apply fuzzy logic principles to AHP, utilizing triangular fuzzy numbers (TFN's) to model pair-wise comparisons and employing the logarithmic least squares method to determine fuzzy weights. This concept was further refined and applied by numerous researchers, including Buckley in 1985 and Chang in 1996, as well as Leung and Cao in 2000. Torra (2010) introduced the concept of hesitancy in Fuzzy sets, which offers the advantage of handling imprecision when multiple sources of vagueness coexist. In recent times, many authors have incorporated hesitant Fuzzy sets into their work. For instance, Zhu (2013)

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introduced the notion of a hesitant fuzzy preference relation and proposed a regression-based approach to transform them into a Fuzzy Preference Relation with the highest consistency degree.

1.1 Consistency of Fuzzy PCM

Measuring the consistency of a fuzzy Pair-wise Comparison Matrix (PCM) in terms of fuzzy quantities can be a challenging task. The literature contains several noteworthy approaches in this regard. Buckley (1985) employed Trapezoidal fuzzy numbers to represent comparison ratios, and fuzzy priorities were derived using the geometric mean method. He extended Saaty's (1980) consistency definition from AHP to Fuzzy AHP. Salo (1996) introduced a Linear Programming method for determining fuzzy weights from relative fuzzy ratios, treating them as constraints on the membership values of local priorities. Arbel (1989) initially defined the feasible region of relative weights in terms of linear inequalities, a concept later extended by Salo and Hamalainer (1995). Their proposal suggested that a fuzzy matrix is considered consistent if there exists a set of crisp weights within the feasible region. Leung and Cao (2000) introduced fuzzy consistency by incorporating tolerance deviation in the feasible region's constraints, building upon Salo's (1996) definition. They suggested an auxiliary linear programming approach to test consistency and proposed an algorithm for deriving fuzzy weights from a consistent matrix. Ramík and Korviny (2010) introduced a new consistency index based on the distance between the matrix and a special ratio matrix, measured by a particular metric. They specified a two-step procedure using metric functions like logarithmic least squares and Chebychev, deriving associated weights. However, Brunelli (2011) pointed out limitations in the consistency index introduced by Ramík and Korviny (2010), noting that the use of the Chebychey metric may fail to capture inconsistency in the Pair-wise Comparison Matrix (PRM). Herrera-Viedma et al. (2004) introduced a new characterization of the consistency property based on the additive transitivity property of fuzzy preference relations. They proposed a method for constructing consistent fuzzy preference relations from a set of (n-1) preference data using this new characterization. They also extended the study of consistency to multiplicative preference relations.

This approach required (n-1) pairwise comparisons for consistent ranking. Liu (2009) defined an acceptably consistent interval reciprocal comparison matrix, which can be achieved by converting intervals into exact numbers. An interval reciprocal comparison matrix with unacceptable consistency can be adjusted to possess acceptable consistency through a convex combination method. This yields a family of crisp reciprocal comparison matrices with acceptable consistency. A formula of possibility degree was presented for ranking interval weights. Liu et al. (2014) introduced a definition of consistent triangular fuzzy reciprocal preference relations based on the reciprocity property. They proposed a method for obtaining consistent triangular fuzzy reciprocal preference relations using (n-1) pairwise comparisons and addressed shortcomings in a proof procedure given by Wang and Chen (2008). Wang et al. (2005) developed a pragmatic method for consistency testing in interval comparison matrices. They used linear programming to derive consistent interval weights from consistent interval comparison matrices and provided an eigenvector-based nonlinear programming approach for generating interval weights when the matrix is inconsistent. A preference ranking method was utilized for comparing interval weights of criteria or ranking alternatives. Cuiping et al. (2008) introduced a method to test the consistency of a fuzzy comparison matrix using the kernels of fuzzy numbers. They proposed a mathematical programming model to assess the matrix's consistency. Bulut et al. (2012) presented a generic version of the conventional Fuzzy-Analytic Hierarchy Process (FAHP) and applied it to the shipping asset management (SAM) problem in the dry bulk shipping market. Their Generic Fuzzy-AHP (GF-AHP) model aimed to ensure consistency within the PCM for expert groups. Kinay and Tezel (2021) used different ranking methods in Fuzzy-AHP for solving problem related Turkish textile company. For more comprehensive information, one can refer to the review article by Liu et al. (2020).

Certain mathematical operations, such as multiplication and inversion applied to Triangular Fuzzy Numbers (TFNs) or Trapezoidal Fuzzy Numbers (TrFNs), do not yield TFNs or TrFNs as results. However, the outcomes closely resemble corresponding fuzzy numbers of their respective types. Despite this, these operations find applications in various fields, including linear programming and decision theory. In the context of Analytic Hierarchy Process (AHP), where numerical judgments are fuzzified, the solutions may not always be real, as indicated by Saaty in 2007. Nevertheless, experts' judgments can often be vague or uncertain, making fuzziness an unavoidable aspect of real-world situations. Despite recognizing the limitations of fuzzy number operations and the demand for real-time solutions, it is advisable to minimize the use of fuzzy operations whenever possible. However, it's worth noting that the initial input data may still be inherently fuzzy.

The remainder of this article is structured as follows: Section 2 provides fundamental definitions related to fuzzy AHP. Section 3 illustrates the proposed method for testing consistency through Geometric Defuzzification with examples. Section 4 is dedicated to enhancing consistency through the proposed algorithm. In Section 5, the application of the proposed method to a problem involving an automobile manufacturer, NEKYEK, is presented, followed by the concluding remarks.

2. PRELIMINARIES

Saaty (1980) introduced a fundamental 9-point scale for comparing the alternatives in AHP. The PCM which are discussed in this paper are all based on this scale only.

Definition 1*:* A matrix $A = \left(a_{ij}\right)_{nxn}$is said to be PCM if a_{ij} indicates the relative preference of i^{th} alternative over j^{th} alternative and satisfies $a_{ij} = \frac{1}{a_{ji}}\ \forall\ i, j.$

Definition 2: A PCM $A = \left(a_{ij}\right)_{nxn}$ is said to be consistent if $a_{ij} = a_{ik}.a_{kj}\ \forall\ i, j\ and\ k.$

Definition 3: Consistency Ratio (CR) of a PCM $A = \left(a_{ij}\right)_{nxn}$ is defined to be

$$CR = \frac{CI}{RI}$$

Where CI is Consistency Index and calculated by

$$CI = \frac{\lambda_{max}(A) - n}{n - 1}$$

And RI is the average Random Index which depends on order of the matrix [page 19 of Saaty (1980)]. According to Saaty (1980), a PCM is said to be consistent if $CR \leq 0.1$.

Definition 4: Let X be a universe of discourse. A fuzzy set $\tilde{A}$ is defined by $\tilde{A} = \left\{\left(x, \mu_{\tilde{A}}(x)\right), X \in x\right\}$ and $\mu_{\tilde{A}}(x): X \to [0,1]$. Here $\mu_{\tilde{A}}(x)$ is called membership function.

Definition 5: A fuzzy set $\tilde{A}$ is said to be convex if $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$ for all $x_1, x_2 \in X$ and $\lambda \in [0,1]$.

Definition 6: A fuzzy set $\tilde{A}$ of the universe of discourse X is said to be normal if there exists a $x_i \in X$ satisfying $\mu_{\tilde{A}}(x_i) = 1$.

Definition 7: A Fuzzy set which is both convex and normal is called Fuzzy Number.

The most used fuzzy numbers are Trapezoidal Fuzzy Number (TrFN) and Triangular Fuzzy Number (TFN), these are respectively defined as follows:

Definition 8: A Trapezoidal Fuzzy Number (TrFN) is denoted by an ordered quadruple as $\widetilde{N} = (a, b, c, d)$ whose membership function $\mu_{\widetilde{N}}(x)$ described as

$$\mu_{\widetilde{N}}(x) = \begin{cases} \frac{(x-a)}{(b-a)}, & a \leq x \leq b \\ 1, & b \leq x \leq c \\ \frac{(d-x)}{(d-c)}, & c \leq x \leq d \\ 0, & otherwise \end{cases}$$

Definition 9: A Triangular Fuzzy Number (TFN) denoted by an ordered triple as $\widetilde{M} = (a, b, c)$ whose membership function $\mu_{\widetilde{M}}(x)$ is described as

$$\mu_{\widetilde{M}}(x) = \begin{cases} \frac{(x-a)}{(b-a)}, & a \leq x \leq b \\ \frac{(d-x)}{(d-b)}, & b \leq x \leq d \\ 0, & otherwise \end{cases}$$

Definition 10: A PCM $\tilde{A} = \left(\tilde{a}_{ij}\right)$ where $\tilde{a}_{ij}$ is a fuzzy number, is called Fuzzy PCM

In the context of Fuzzy AHP, the comparison of two fuzzy numbers involves the utilization of either Ranking methods or Defuzzification methods. While many ranking methods may yield a crisp value that falls outside the range of the original fuzzy number, defuzzification methods ensure that the obtained value aligns with the original fuzzy number's range.

Numerous defuzzification methods exist within Fuzzy Set Theory. Nonetheless, in the realm of Fuzzy AHP, two specific methods are commonly employed by many researchers in their studies. Weighted Mean (WM) defuzzification of a TFN $\widetilde{N} = (l, m, u)$ is defined by the following equation

$$WM(\widetilde{N}) = \frac{l + 4m + u}{6}$$

The Centroid Method, also known as the Center of Gravity Method, stands out as one of the most frequently employed techniques among all defuzzification methods. If a TFN $\widetilde{N}$ is linked to the membership function $\mu_{\widetilde{N}}(x)$, the Centroid Defuzzification is obtained by the centre of gravity formula, expressed by the following equation.

$$CM(\widetilde{N}) = \frac{\int \mu_{\widetilde{N}}(x).x\,dx}{\int \mu_{\widetilde{N}}(x)\,dx}$$

It can be shown easily that $CM(\widetilde{N})$ of a TFN $\widetilde{N} = (l, m, u)$ is

$$CM(\widetilde{N}) = \frac{l + m + u}{3}$$

It is noticeable that the defuzzified value associated with the inverse of a Triangular Fuzzy Number (TFN) does not coincide with the inverse of the defuzzified value obtained through the aforementioned defuzzification methods. Consequently, these methods may not be suitable in Analytic Hierarchy Process (AHP), where the elements in the Pairwise Comparison Matrix (PCM) adhere to the reciprocity rule. Thus, there is a need to establish a new defuzzification approach that aligns with the reciprocity rule.

3. CONSISTENCY THROUGH GEOMETRIC DEFUZZIFICATION

3.1 Geometric Defuzzyfication (GD)

Geometric Defuzzyfication (GD) of a TFN $\widetilde{N} = (l, m, u)$ is defined to be geometric mean of l, m, u or it is defined by the equation

$$GD(\widetilde{N}) = (l * m * u)^{1/3}$$

Similarly, GD of a trapezoidal fuzzy number $\tilde{T} = (a, b, c, d)$ is defined by the equation

$$GD(\tilde{T}) = (a * b * c * d)^{1/4}$$

And GD of an interval fuzzy number $\tilde{I} = [l,\ u]$ is defined by the equation

$$GD(\tilde{I}) = (l * u)^{1/2}$$

Remark: Here it can be noted that $GD(\widetilde{N}^{-1}) = \frac{1}{GD(\widetilde{N})}$.

By applying Geometric Defuzzification (GD) to each element in the Fuzzy Pairwise Comparison Matrix (Fuzzy PCM), a corresponding crisp matrix, which is a Pairwise Comparison Matrix (PCM), can be obtained. Similarly, employing the above defuzzification method yields a crisp PCM corresponding to a Fuzzy PCM with trapezoidal fuzzy numbers or an interval PCM.

3.2 Consistency of a Fuzzy PCM:

Definition 11: A fuzzy PCM $\tilde{A} = \{\tilde{a}_{ij}\}_{nxn}$, where $\tilde{a}_{ij}$ is anyone of Triangular Fuzzy Number, Trapezoidal fuzzy number and interval, is said to be consistent if and only if $G_{\tilde{A}} = \{GD(\tilde{a}_{ij})\}_{nxn}$ is consistent.

Example 1:

Consider a fuzzy PCM $\tilde{A}$ whose elements are TFNs which was used in the study of student's requirement problem by Kamvysi et al (2014). The elements in the matrix $\tilde{A} = (\tilde{a}_{ij})_{4x4}$ describe the comparisons, made among the factors Theory-based Knowledge, Practical-based Knowledge and Generic Academic Skills Key Transferable Skills.

$\tilde{a}_{11} = \tilde{a}_{22} = \tilde{a}_{33} = \tilde{a}_{44} = (1, 1, 1)$
$\tilde{a}_{12} = (0.4052,0.467,0.6904)$
$\tilde{a}_{13} = (0.3604,0.4328,0.5957)$
$\tilde{a}_{14} = (0.3392,0.3992,0.5557)$
$\tilde{a}_{21} = (1.448,2.1413,2.4672)$
$\tilde{a}_{23} = (1.0185,1.145,1.8737)$
$\tilde{a}_{24} = (0.8112,0.9369,1.4618)$
$\tilde{a}_{31} = (1.6787,2.3105,2.7747)$
$\tilde{a}_{32} = (0.5337,0.8734,0.9818)$
$\tilde{a}_{34} = (0.6955,0.7959,1.3044)$
$\tilde{a}_{41} = (1.7995,2.5050,2.5050)$
$\tilde{a}_{42} = (0.6841,1.0674,1.2327)$
$\tilde{a}_{43} = (0.7666,1.2564,1.4378)$

Using GD, $\tilde{A}$ is converted to crisp PCM $G_{\tilde{A}} = \{GD(\tilde{a}_{ij})\}$ which is

$$G_{\tilde{A}} = \begin{pmatrix} 1 & 0.5074 & 0.4529 & 0.4222 \\ 1.9708 & 1 & 1.2976 & 1.0357 \\ 2.2078 & 0.7706 & 1 & 0.8971 \\ 2.2435 & 0.9655 & 1.1147 & 1 \end{pmatrix}$$

The CR of $G_{\tilde{A}}$ is 0.0, which means $G_{\tilde{A}}$ is consistent and thus by *Definition 11*, $\tilde{A}$ is consistent. Kamvysi et al (2014) used the degree of optimism technique to convert the PCM $\tilde{A}$ into corresponding crisp PCM and is given by

$$\begin{pmatrix} 1 & 0.5478 & 0.478 & 0.4475 \\ 1.9583 & 1 & 1.4461 & 1.1365 \\ 2.2267 & 0.7578 & 1 & 0.9999 \\ 2.3737 & 0.9584 & 1.1023 & 1 \end{pmatrix}$$

The CR of the above crisp PCM is 0.052 and hence it is consistent.

Example 2:

Consider a fuzzy PCM $\tilde{B} = (\tilde{b}_{ij})_{4x4}$ with Trapezoidal Fuzzy Numbers which was used for consistency test proposed by Cuiping et al (2008).

$\tilde{b}_{11} = \tilde{b}_{22} = \tilde{b}_{33} = (1, 1, 1,1)$
$\tilde{b}_{12} = (1,2,5,6)$
$\tilde{b}_{13} = (1,2,4,5)$
$\tilde{b}_{14} = (1/2,1,3,7/2)$
$\tilde{b}_{21} = (1/6,1/5,1/2,1)$
$\tilde{b}_{23} = (1/2,1,3,7/2)$
$\tilde{b}_{24} = (1/2,1,2,5/2)$
$\tilde{b}_{31} = (1/5,1/4,1/2,1)$
$\tilde{b}_{32} = (2/7,1/3,1,2)$
$\tilde{b}_{34} = (1/4,1/2,1,3/2)$
$\tilde{b}_{41} = (2/7,1/3,1,2)$
$\tilde{b}_{42} = (2/5,1/2,1,2)$
$\tilde{b}_{43} = (2/3,1,2,4)$

The corresponding crisp PCM $G_{\tilde{B}} = \{GD(\tilde{b}_{ij})\}$ using definition of GD is given by

$$G_{\tilde{B}} = \begin{pmatrix} 1 & 2.7831 & 2.5148 & 1.5137 \\ 0.3593 & 1 & 1.5137 & 1.2574 \\ 0.39763 & 0.66063 & 1 & 0.65804 \\ 0.66063 & 0.79527 & 1.5196 & 1 \end{pmatrix}$$

The CR of $G_{\tilde{B}}$is 0.0241, thus $G_{\tilde{B}}$ is consistent and hence $\tilde{B}$ is consistent. Cuiping et al (2008) used α- cut method to convert $\tilde{B}$ into corresponding Interval matrix and hence shown that it is consistent by means of kernels of the fuzzy numbers.

Example 3

Consider an interval PCM $\tilde{C}$. This PCM is used by Wang et al (2005) to describe their proposed consistency test.

$$\tilde{C} = \begin{pmatrix} [1,1] & [2,5] & [2,4] & [1,3] \\ [1/5,1/2] & [1,1] & [1,3] & [1,2] \\ [1/4,1/2] & [1/3,1] & [1,1] & [1/2,1] \\ [1/3,1] & [1/2,1] & [1,2] & [1,1] \end{pmatrix}$$

Using GD, one can get the corresponding crisp PCM $G_{\tilde{C}}$ of $\tilde{C}$ as

$$G_{\tilde{C}} = \begin{pmatrix} 1 & 3.162 & 2.828 & 1.732 \\ 0.3162 & 1 & 1.732 & 1.414 \\ 0.3535 & 0.5773 & 1 & 0.7071 \\ 0.5773 & 0.7071 & 1.414 & 1 \end{pmatrix}$$

The CR of $G_{\tilde{C}}$ is 0.0323, which implies that $G_{\tilde{C}}$ and thus $\tilde{C}$ is also consistent. Wang et al (2005) has also shown that $\tilde{C}$ is consistent. Here, one more observation is that the interval matrix corresponding to $\tilde{B}$ by α- cut method is $\tilde{C}$. Moreover $\tilde{C}$ was also examined by Arbel and Vergas (1993) for consistency.

4. CONSISTENCY IMPROVEMENT METHOD

At times, the judgment matrix provided by experts may exhibit inconsistency, necessitating adjustments either by the experts or the decision maker to alleviate this inconsistency. The following algorithm is suggested to enhance the consistency of a Fuzzy PCM.

Algorithm

Step 1: Input the PCM $\tilde{A} = \{\widetilde{a_{ij}}\}_{nxn}$, where $\widetilde{a_{ij}}$ is anyone of TFN, TrFN and Interval number.
Step 2: Find the corresponding crisp PCM $G_{\tilde{A}} = \{GD(\widetilde{a_{ij}})\}_{nxn}$ using Geometric Defuzzification (GD).
Step 3: If CR of $G_{\tilde{A}} \leq 0.1$ then print '$\tilde{A}$' is consistent. STOP
Step 4: If CR of $G_{\tilde{A}} > 0.1$ then

Compute
$G_{\hat{A}} = [\hat{a}_{ij}]$ where $\hat{a}_{ij} = \frac{\sum_k a_{ik}}{\sum_k a_{jk}}$
$e_{ij}^2 = \frac{(a_{ij} - \hat{a}_{ij})^2}{\hat{a}_{ij}}$ and $\chi^2 = \sum \sum e_{ij}^2$
Pick i^* and j^* for which e_{ij}^2 is the maximum.
Replace $a_{i^*j^*}$ by $a_{ij} = \frac{\sum_{k \neq j^*} a_{ik} - 1}{\sum_{k \neq i^*} a_{jk} - 1}$
Go to Step 3.

This method can be explained through the following example.

Example 4

Consider a fuzzy PCM $\widetilde{D} = (\tilde{d}_{ij})_{5x5}$ with TrFNs as pair wise comparisons, which was used by Ghazanfari and Nojavan (2004) for describing their method of reducing inconsistency.

$\tilde{d}_{11} = \tilde{d}_{22} = \tilde{d}_{33} = \tilde{d}_{44} = \tilde{d}_{55} = (1, 1, 1,1)$
$\tilde{d}_{12} = (1,2,4,5)$ $\quad \tilde{d}_{13} = \left(\frac{1}{9}, \frac{1}{8}, \frac{1}{7}, \frac{1}{6}\right)$
$\tilde{d}_{14} = \left(\frac{1}{9}, \frac{1}{7}, \frac{1}{6}, \frac{1}{5}\right)$ $\quad \tilde{d}_{15} = \left(\frac{1}{8}, \frac{1}{7}, \frac{1}{6}, \frac{1}{5}\right)$
$\tilde{d}_{21} = \left(\frac{1}{5}, \frac{1}{4}, \frac{1}{2}, 1\right)$ $\quad \tilde{d}_{23} = \left(\frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}\right)$
$\tilde{d}_{24} = \left(\frac{1}{7}, \frac{1}{6}, \frac{1}{6}, \frac{1}{5}\right)$ $\quad \tilde{d}_{25} = \left(\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, 1\right)$
$\tilde{d}_{31} = (6,7,8,9)$ $\quad \tilde{d}_{32} = (9,9,9,9)$
$\tilde{d}_{34} = (1,2,4,5)$ $\quad \tilde{d}_{35} = (1,2,3,4)$
$\tilde{d}_{41} = (5,6,7,9)$ $\quad \tilde{d}_{42} = (5,6,6,7)$
$\tilde{d}_{43} = \left(\frac{1}{5}, \frac{1}{4}, \frac{1}{2}, 1\right)$ $\quad \tilde{d}_{45} = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right)$
$\tilde{d}_{51} = (5,6,7,8)$ $\quad \tilde{d}_{52} = (1,2,2,4)$
$\tilde{d}_{53} = \left(\frac{1}{4}, \frac{1}{3}, \frac{1}{2}, 1\right)$ $\quad \tilde{d}_{54} = (2,2,3,3)$

The crisp PCM $G_{\widetilde{D}}$ corresponding to Fuzzy PCM $\widetilde{D}$ using GD is

$$G_{\widetilde{D}} = \begin{pmatrix} 1 & 2.5148 & 0.13485 & 0.15166 & 0.15619 \\ 0.39763 & 1 & 0.11111 & 0.16784 & 0.5 \\ 7.4156 & 9 & 1 & 2.5148 & 2.2133 \\ 6.5935 & 5.9579 & 0.39763 & 1 & 0.40825 \\ 6.4022 & 2 & 0.4518 & 2.4495 & 1 \end{pmatrix}$$

The CR of $G_{\widetilde{D}}$ is 0.127, thus it can be considered as inconsistent and hence corresponding fuzzy PCM $\widetilde{D}$ is also inconsistent.
Using the proposed algorithm, the modified matrix $G_{\widetilde{D}}^m$ can be obtained as

$$G_{\widetilde{D}}^m = \begin{pmatrix} 1.0000 & 2.5149 & 0.1349 & 0.1517 & 0.5716 \\ 0.3976 & 1.0000 & 0.1111 & 0.1678 & 0.5000 \\ 7.4156 & 9.0000 & 1.0000 & 2.5149 & 2.2134 \\ 6.5935 & 5.9579 & 0.3976 & 1.0000 & 3.0821 \\ 1.7496 & 2.0000 & 0.4518 & 0.3245 & 1.0000 \end{pmatrix}$$

The CR of $G_{\widetilde{D}}^m$ is 0.0514, thus it can be considered as consistent. This modified matrix obtained by two iterations only with changing the elements $G_{\widetilde{D}_{15}}$, $G_{\widetilde{D}_{45}}$ and their reciprocals.

Ghazanfari and Nojavan (2004) also shown that $\widetilde{D}$ is inconsistent by using penalty function and the changeable preference mathematical programming model have been used to reduce the inconsistency wherein six judgments ($\widetilde{D}_{12}, \widetilde{D}_{41}, \widetilde{D}_{42}, \widetilde{D}_{51}, \widetilde{D}_{52}$ and $\widetilde{D}_{54}$) and their reciprocals are modified.

In this context, it is evident that the revised, consistent crisp Pairwise Comparison Matrix (PCM) is presented rather than a Fuzzy PCM corresponding to an inconsistent Fuzzy PCM. There is no need to transform the consistent crisp PCM into a Fuzzy PCM, as both result in the same preference order. This can be elucidated further through the following numerical example.

5. AN AUTOMOBILE MANUFACTURER NEKYEK PROBLEM

Within the context of an automobile manufacturing company, a challenge known as the NEKYEK problem has been the subject of investigation by Kahraman et al (2003), Wang and Chen (2008), and Liu et al (2014), who have documented their respective approaches. The NEKYEK problem revolves around the decision-making process for establishing a new factory and determining the optimal location from a range of options. This decision hinges on several criteria, namely Environmental Regulation (E), Host Community (H), Competitive Advantage (C), and Political Risk (P). The available location choices are Istanbul (A1), Ankara (A2), and Izmir (A3). The hierarchical structure of this problem is illustrated in Figure 1

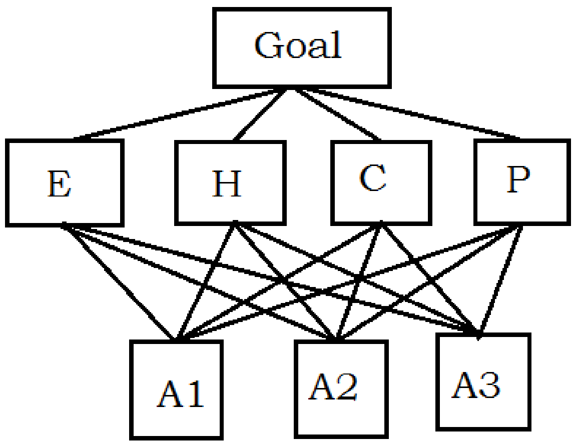

Figure 1. Hierarchy structure of Automobile manufacturer NEKYEK

The pairwise comparisons for each level that is four criteria E, H, C and P under goal and the alternatives A1, A2 and A3 under each criterion are given as Triangular fuzzy numbers by a decision maker and the corresponding PCMs are given. The fuzzy PCM for criteria under goal is

$\widetilde{M}^{Cri} = \left(\widetilde{m}_{ij}\right)_{4x4}$ where

$$\widetilde{m}_{11} = \widetilde{m}_{22} = \widetilde{m}_{33} = \widetilde{m}_{44} = (1,1,1)$$
$$\widetilde{m}_{12} = \left(\frac{3}{2},2,\frac{5}{2}\right) \qquad \widetilde{m}_{13} = \left(\frac{2}{7},\frac{1}{3},\frac{2}{5}\right)$$
$$\widetilde{m}_{14} = \left(\frac{5}{2},3,\frac{7}{2}\right) \qquad \widetilde{m}_{21} = \left(\frac{2}{5},\frac{1}{2},\frac{2}{3}\right)$$
$$\widetilde{m}_{23} = \left(\frac{2}{7},\frac{1}{3},\frac{2}{5}\right) \qquad \widetilde{m}_{24} = \left(\frac{7}{2},4,\frac{9}{2}\right)$$
$$\widetilde{m}_{31} = \left(\frac{5}{2},3,\frac{7}{2}\right) \qquad \widetilde{m}_{32} = \left(\frac{5}{2},3,\frac{7}{2}\right)$$
$$\widetilde{m}_{34} = \left(\frac{5}{2},3,\frac{7}{2}\right) \qquad \widetilde{m}_{41} = \left(\frac{2}{7},\frac{1}{3},\frac{2}{5}\right)$$
$$\widetilde{m}_{42} = \left(\frac{2}{9},\frac{1}{4},\frac{2}{7}\right) \qquad \widetilde{m}_{43} = \left(\frac{2}{7},\frac{1}{3},\frac{2}{5}\right)$$

The fuzzy PCMs for alternatives under all criteria are

$$\widetilde{M}^{E} = \begin{pmatrix} (1,1,1) & (2/5,1/2,2/3) & (2/5,1/2,2/3) \\ (3/2,2,5/2) & (1,1,1) & (1/2,2/3,1) \\ (3/2,2,5/2) & (1,3/2,2) & (1,1,1) \end{pmatrix}$$

$$\widetilde{M}^{H} = \begin{pmatrix} (1,1,1) & (2/5,1/2,2/3) & (1/2,2/3,1) \\ (3/2,2,5/2) & (1,1,1) & (1,3/2,2) \\ (1,3/2,2) & (1/2,2/3,1) & (1,1,1) \end{pmatrix}$$

$$\widetilde{M}^{C} = \begin{pmatrix} (1,1,1) & (1/2,2/3,1) & (2/5,1/2,2/3) \\ (1,3/2,2) & (1,1,1) & (1/2,1,3/2) \\ (3/2,2,5/2) & (2/3,1,2) & (1,1,1) \end{pmatrix}$$

$$\widetilde{M}^{P} = \begin{pmatrix} (1,1,1) & (2/5,1/2,2/3) & (1/2,2/3,1) \\ (3/2,2,5/2) & (1,1,1) & (1,3/2,2) \\ (1,3/2,2) & (1/2,2/3,1) & (1,1,1) \end{pmatrix}$$

Using the GD, it can be verified the consistency of fuzzy PCMs. The crisp PCM $G_{\widetilde{M}^{Cri}}$ of the fuzzy PCM $\widetilde{M}^{Cri}$ is

$$G_{\widetilde{M}^{Cri}} = \begin{pmatrix} 1.0000 & 1.9574 & 0.3365 & 2.9720 \\ 0.5109 & 1.0000 & 0.3365 & 3.9791 \\ 2.9720 & 2.9720 & 1.0000 & 2.9720 \\ 0.3365 & 0.2513 & 0.3365 & 1.0000 \end{pmatrix}$$

The CR of $G_{\widetilde{M}^{Cri}}$ is 0.1064 which means that $G_{\widetilde{M}^{Cri}}$ is inconsistent and correspondingly $\widetilde{M}^{Cri}$ is also inconsistent. Thus, it requires modification.

Using Algorithm, the modified consistent matrix $G^{m}_{\widetilde{M}^{Cri}}$ is obtained as

$$G^{m}_{\widetilde{M}^{Cri}} = \begin{pmatrix} 1.0000 & 1.9574 & 0.8293 & 2.9720 \\ 0.5109 & 1.0000 & 0.3365 & 3.9791 \\ 1.2058 & 2.9720 & 1.0000 & 2.9720 \\ 0.3365 & 0.2513 & 0.3365 & 1.0000 \end{pmatrix}$$

The CR of $G^{m}_{\widetilde{M}^{Cri}}$ is 0.0698. It is obtained only by one iteration i.e., modifying the element ${G_{\widetilde{M}^{Cri}}}_{13}$ and its reciprocal. The priority vector obtained from $G^{m}_{\widetilde{M}^{Cri}}$ which means the priority weights of criteria by Eigen Vector Method (EVM) is

$$S_{G^{m}_{\widetilde{M}^{Cri}}} = (0.3121 \quad 0.2044 \quad 0.3930 \quad 0.0905)$$

Similarly, the consistency of the fuzzy PCMs can be verified through their respective crisp PCMs. The crisp PCMs of the fuzzy PCMs $\widetilde{M}^{E}$, $\widetilde{M}^{H}$, $\widetilde{M}^{C}$ and $\widetilde{M}^{P}$ with CRs are

$$G_{\widetilde{M}^{E}} = \begin{pmatrix} 1.0000 & 0.5109 & 0.5109 \\ 1.9574 & 1.0000 & 0.6934 \\ 1.9574 & 1.4422 & 1.0000 \end{pmatrix}$$

$$G_{\widetilde{M}^{H}} = \begin{pmatrix} 1.0000 & 0.5109 & 0.6934 \\ 1.9574 & 1.0000 & 1.4422 \\ 1.4422 & 0.6934 & 1.0000 \end{pmatrix}$$

$$G_{\widetilde{M}^{C}} = \begin{pmatrix} 1.0000 & 0.6934 & 0.5109 \\ 1.4422 & 1.0000 & 0.9086 \\ 1.9574 & 1.1006 & 1.0000 \end{pmatrix}$$

$$G_{\widetilde{M}^{P}} = \begin{pmatrix} 1.0000 & 0.5109 & 0.6934 \\ 1.9574 & 1.0000 & 1.4422 \\ 1.4422 & 0.6934 & 1.0000 \end{pmatrix}$$

The CRs of $G_{\widetilde{M}^{E}}$, $G_{\widetilde{M}^{H}}$, $G_{\widetilde{M}^{C}}$ and $G_{\widetilde{M}^{P}}$ are respectively 0.0128, 0, 0.0042 and 0. Thus the matrices $G_{\widetilde{M}^{E}}$, $G_{\widetilde{M}^{H}}$,

$G_{\widetilde{M}^C}$ and $G_{\widetilde{M}^P}$ are consistent and so their Fuzzy PCMs $\widetilde{M}^E$, $\widetilde{M}^H$, $\widetilde{M}^C$ and $\widetilde{M}^P$are also consistent. Thus, there will be modification not required. The priority vectors of alternatives (A1, A2, and A3) under the criteria Environmental regulation (E), Host community (H), Competitive advantage (C) and Political risk (P) are obtained from $G_{\widetilde{M}^H}$, $G_{\widetilde{M}^C}$ and $G_{\widetilde{M}^P}$ by EVM are

$$S_{G_{\widetilde{M}^E}} = (0.2023 \quad 0.3504 \quad 0.4473)$$
$$S_{G_{\widetilde{M}^H}} = (0.2267 \quad 0.4529 \quad 0.3204)$$
$$S_{G_{\widetilde{M}^C}} = (0.2287 \quad 0.3538 \quad 0.4175)$$
$$S_{G_{\widetilde{M}^P}} = (0.2267 \quad 0.4529 \quad 0.3204)$$

The global weights of the alternatives can be obtained by taking

$$(0.3121 \quad 0.2044 \quad 0.3930 \quad 0.0905).\begin{pmatrix} 0.2023 & 0.3504 & 0.4473 \\ 0.2267 & 0.4529 & 0.3204 \\ 0.2287 & 0.3538 & 0.4175 \\ 0.2267 & 0.4529 & 0.3204 \end{pmatrix} = (0.2199 \quad 0.3820 \quad 0.3982)$$

Thus, the final rank order of the alternatives is $A3 \succ A2 \succ A1$. This rank order of the alternatives coincides with that of the rank order obtained by methods of Kahraman et al (2003), Wang and Chen (2008) and Liu et al (2014).

6. CONCLUSIONS

In this article a new defuzzification method named, Geometric Defuzzification (GD) of a fuzzy number, is defined. A crisp PCM can be obtained corresponding to fuzzy PCM by GD of elements in fuzzy PCM. New definition of consistency for fuzzy PCM has been introduced by means corresponding crisp PCM. The properties of new definition have been discussed. A simple algorithm is proposed and utilized to improve the consistency of some numerical examples which are widely studied in the literature. Moreover, the validity of the proposed method has been addressed by applying it to an automobile manufacturer NEKYE problem.

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