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ON STEREOGRAPHIC SEMICIRCULAR ERLANG DISTRIBUTION WITH APPLICATION

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A B S T R A C T

In this research paper, we present an innovative investigation into a novel two parameter semicircular distribution, termed the "stereographic semicircular Erlang distribution," which is constructed using the inverse stereographic projection (ISP) technique. This distribution serves as advancement over the existing stereographic semicircular exponential distribution. We delve into essential mathematical properties of this distribution and execute a simulation study to estimate its parameter values. Furthermore, we perform an empirical analysis utilizing a dataset comprising posterior corneal curvature measurements extracted from the eyes of 23 patients. This empirical assessment is designed to evaluate the adaptability and potential applicability of the proposed distribution within the realm of ophthalmology in medical science.

1. INTRODUCTION

Circular data find widespread applications across various disciplines such as geology, meteorology, biology, earth science, political science, economics, and computer science, among others. Full circular models are extensively documented in seminal texts, including Fisher (1993), Mardia and Jupp (2000), and Jammalamadaka and Sen Gupta (2001). Nevertheless, it is essential to recognize that modeling circular data across the entire circle may not always be necessary, as acknowledged by Jones (1968), Guardiola (2004), Byoung et al. (2008), Phani et al. (2013, 2016, 2017, 2017a, 2019, 2020), and Girija et al. (2013). Noteworthy contributions have been made by Dattatreya Rao et al. (2007), Phani et al. (2011, 2012, 2023), Sakthivel et al. (2022), Oleiwi et al. (2022), and Salah Hamza Abid (2022, 2023) have introduced various circular and semicircular models through the application of inverse stereographic projection, a technique that maps point from the real line to the unit circle based on known probability distributions on real line. Further enriching this field Pramesti et al. (2015, 2016, 2017, and 2018) have explored and analyzed novel semicircular and circular distributions. Recent research by Rambli et al. (2015), Ali (2017) and Iftikhar et al.

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(2022) has introduced half circular distributions and discussed their applicability to real-world data sets.

In this article, we initiate the development of a novel semicircular model, herein denominated as the “stereographic semicircular Erlang distribution.” This model is constructed by applying the inverse stereographic projection technique to the Erlang distribution and representing a special case of the gamma distribution. We furnish precise mathematical representations for trigonometric moments using Meijer's G-function. The structure of the article unfolds as follows: In the second section, we introduce the stereographic semicircular Erlang distribution and present key mathematical properties, including trigonometric moments, cumulative distribution function, survival function, and hazard rate function, accompanied by illustrative plots. Section 3 outlines the method of maximum likelihood estimation, followed by a simulation study in section 4 to assess the model’s parameter consistency. Section 5 scrutinizes the applicability of the new model to real-world datasets, offering comparisons with other competitive models. Finally, in section 6, we summarize our findings and conclude this piece of work.

2. DERIVATION OF THE PROPOSED MODEL

The Erlang distribution, originally introduced by Erlang (1909), is a particular instance of the gamma distribution, characterized by a positive integer value for the shape parameter. It is a continuous probability distribution with support on $(0,\infty)$, and has wide range of applications in fields like traffic engineering , stochastic processes and biomathematics, mainly due to its relative to the exponential distribution.

Here we recall the definition of Erlang distribution.

Definition 2.1

A continuous random variable X is considered to adhere to the Erlang distribution with a shape parameter k (a positive integer) and a scale parameter $\lambda > 0$ if its probability density and distribution functions are defined as follows:

$$f(x)=\frac{\lambda^k x^{k-1}}{\Gamma(k)}e^{-\lambda x},\ \text{where } k\in Z^+,\lambda>0,0<x<\infty. \tag{1}$$

$$F(x)=\frac{\gamma(\lambda,\lambda x)}{\Gamma(\lambda)},\ \text{where } \gamma(.) \text{ is the lower incomplete gamma function.} \tag{2}$$

Definition 2.2

A random variable θ_{SC} defined on the semicircle is characterized as following the stereographic semicircular Erlang distribution with a shape parameter k (a positive integer) and a scale parameter λ, denoted by $\mathrm{SSCEr}(k,\lambda)$. This distribution is specified by its probability density and distribution functions, given as follows:

$$g(\theta;k,\lambda)=\lambda^k\left(\Gamma(k)\times(1+\cos((\theta)))\right)^{-1}\times\left(\tan\left(\frac{\theta}{2}\right)\right)^{(-1+k)}\times\exp\left(-\lambda\tan\left(\frac{\theta}{2}\right)\right) \tag{3}$$

$$G(\theta;k,\lambda)=\left(\Gamma(k)\right)^{-1}\times\gamma\left(k,\lambda\tan\left(\frac{\theta}{2}\right)\right),\ \text{where } \theta\in[0,\pi),\lambda>0,\text{ and } k\in Z^+. \tag{4}$$

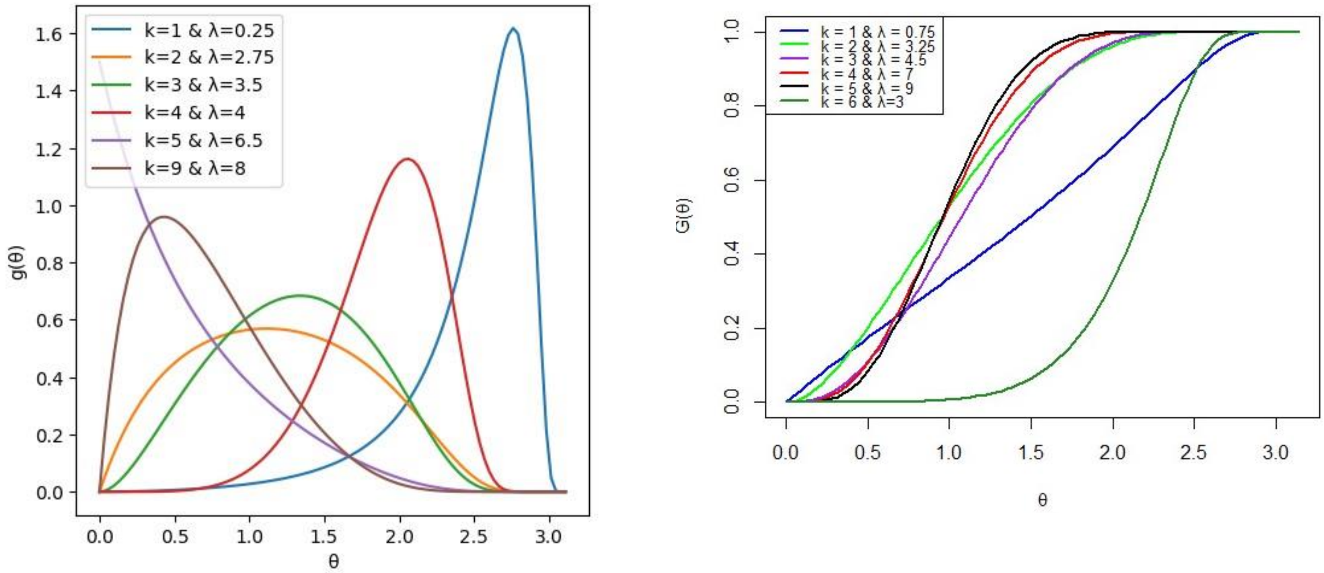

Figure 1. Plots of the probability density function (left) and cumulative distribution function (right) for various parameter values

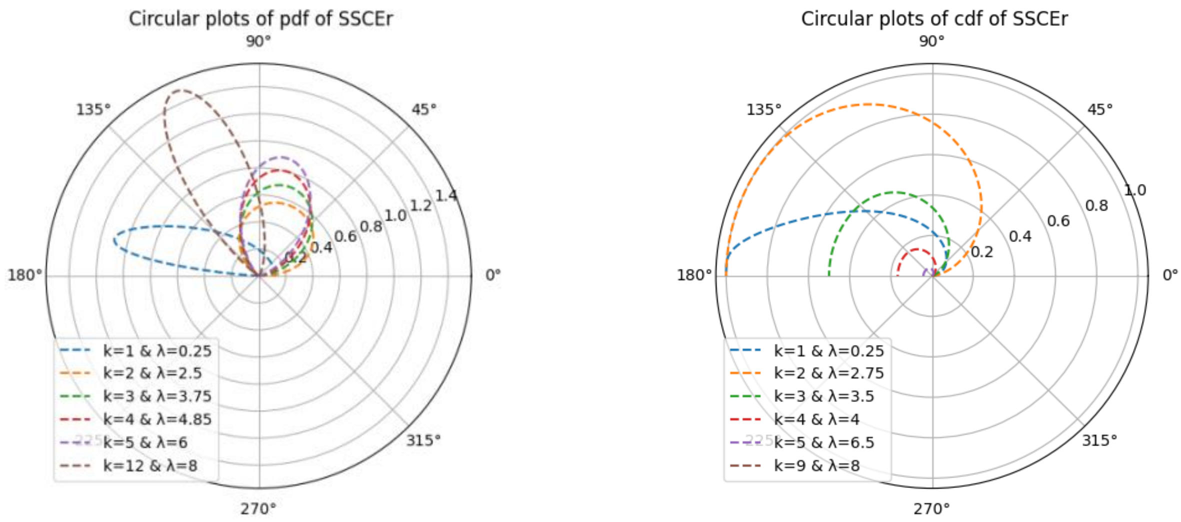

Figure 2. Plots displaying the probability density function (left) and cumulative distribution function (right) for diverse parameter values, represented in a circular format

Survival and Hazard Function: The survival function of SSCEr(k,λ) is given by

$$S(\theta)=1-G(\theta)=1-\gamma\left(k,\lambda\tan\left(\frac{\theta}{2}\right)\right)\times(\Gamma(k))^{-1}. \tag{5}$$

The hazard rate function and the reversed hazard rate function of the SSCEr(k,λ) are given respectively by

$$h(\theta)=\frac{g(\theta)}{1-G(\theta)}$$
$$=\frac{\lambda^k\times\left(\left(\tan\left(\frac{\theta}{2}\right)\right)^{k-1}\exp\left(-\lambda\tan\left(\frac{\theta}{2}\right)\right)\right)}{(1+\cos(\theta))\times\left(\Gamma(k)-\gamma\left(k,\lambda\tan\left(\frac{\theta}{2}\right)\right)\right)} \tag{6}$$

and

$$r(\theta)=\frac{g(\theta)}{G(\theta)}$$
$$=\frac{\lambda^k\times\left(\left(\tan\left(\frac{\theta}{2}\right)\right)^{k-1}\exp\left(-\lambda\tan\left(\frac{\theta}{2}\right)\right)\right)}{\left((1+\cos(\theta))\times\gamma\left(k,\lambda\tan\left(\frac{\theta}{2}\right)\right)\right)} \tag{7}$$

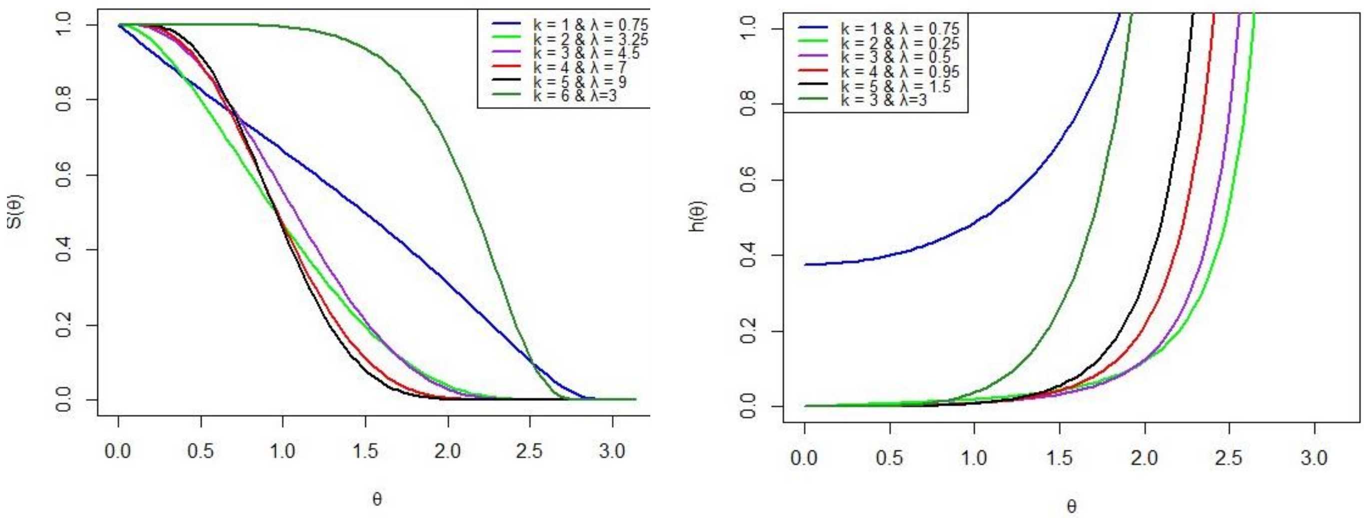

Figure 3. Survival function (left) and hazard rate function (right) plots depicting varying parameter values

Quantile Function: The quantile function for the SSCEr(k,λ) distribution is given by

$$G^{-1}(u)=Q(u)=2\times\tan^{-1}\left(\left(\frac{1}{\lambda}\right)\times\gamma^{-1}\left(k,\Gamma(k)\times u\right)\right),$$

where $0<u<1$ (8)

and $\gamma^{-1}(.,.)$ is the inverse incomplete gamma function.

Median: The median of SSCEr(k,λ) will be given by

$$Q(0.5)=2\tan^{-1}\left(\left(\frac{1}{\lambda}\right)\gamma^{-1}\left(k,\frac{\Gamma(k)}{2}\right)\right). \quad (9)$$

Trigonometric moments

Under the pdf of stereographic semicircular Erlang distribution the first two $\alpha_p=E(\cos p\theta)$ and $\beta_p=E(\sin p\theta)$, $p=1,2.$ are given as follows:

$$\alpha_1=1-\frac{2\lambda^k}{\sqrt{\pi}\,\Gamma(k)}G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-\frac{k}{2}\\ -\frac{k}{2},0,\frac{1}{2}\end{matrix}\right), \quad (10)$$

$$\beta_1=\frac{\lambda^k}{\sqrt{\pi}\,\Gamma(k)}G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}\left(\frac{1-k}{2}\right)\\ \left(\frac{1-k}{2}\right),0,\frac{1}{2}\end{matrix}\right) \quad (11)$$

$$\alpha_2=1-\frac{4\lambda^k}{\sqrt{\pi}\,\Gamma(k)}G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-\frac{k}{2}\\ \left(\frac{2-k}{2}\right),0,\frac{1}{2}\end{matrix}\right) \quad (12)$$

$$\beta_2=\frac{2\lambda^k}{\sqrt{\pi}\,\Gamma(k)}\left[G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}\left(\frac{1-k}{2}\right)\\ \left(\frac{3-k}{2}\right),0,\frac{1}{2}\end{matrix}\right)-G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-\left(\frac{1+k}{2}\right)\\ \left(\frac{1-k}{2}\right),0,\frac{1}{2}\end{matrix}\right)\right] \quad (13)$$

3. MAXIMUM LIKELIHOOD ESTIMATION

In this section, we have introduced the maximum likelihood estimation method, which is utilized for parameter estimation in the Stereographic Semicircular Erlang (SSCEr) distribution. Assuming that a random sample $\psi_1,\psi_2,\psi_3,...,\psi_n$ of size n is drawn from SSCEr, the log-likelihood function can be expressed as follows:

$$\log L=nk\log(\lambda)-n\log(\Gamma(k))+2\sum_{i=1}^{n}\sec\left(\frac{\psi_i}{2}\right)+(k-1)\sum_{i=1}^{n}\log\left(\tan\left(\frac{\psi_i}{2}\right)\right)-\lambda\sum_{i=1}^{n}\left(\tan\left(\frac{\psi_i}{2}\right)\right)$$

For a fixed value of k, we get $\lambda=\dfrac{nk}{\sum_{i=1}^{n}\left(\tan\left(\frac{\psi_i}{2}\right)\right)}$ (14)

4. SIMULATION

In this part, the performance of λ is evaluated by conduction Monte Carlo simulation study. To carry out this study, we use the inverse distribution function approach (i.e., quantile function) for obtaining random numbers from the SSCEr(k,λ) with pdf and cdf given in Eq. (2.3) and (2.4) respectively. For each simulation, 10,000 samples of sizes $n=50,75,100,300,500,$ and 750 were generated for different values of λ and given k. For every individual sample, we use a self-programmed R script to calculate the Maximum Likelihood Estimators (MLEs), average bias, mean square error (MSE), and mean relative error (MRE).

(i) $\text{Average absolute bias}=\frac{1}{10000}\sum_{i=1}^{10000}\left|(\lambda-\lambda)\right|$

(ii) $\text{Mean Square Error}(MSE)=\frac{1}{10000}\sum_{i=1}^{10000}(\lambda-\lambda)^2$

(iii) $\text{Mean Relative Error}(\text{MRE})=\frac{1}{10000}\sum_{i=1}^{10000}\frac{|\lambda-\lambda|}{\lambda}$

Table 1. Average MLE, absolute bias, MSE, and MRE of the simulated estimate of λ for a given value of k

	$k=1$							
	$\lambda=0.75$				$\lambda=2$			
Sample size n	MLE	$\lvert Bias \rvert$	MSE	MRE	MLE	$\lvert Bias \rvert$	MSE	MRE
50	0.76901	0.08783	0.01278	0.11710	2.05177	0.24247	0.09549	0.12123
75	0.76765	0.07406	0.00900	0.09874	2.03321	0.18947	0.06012	0.09474
300	0.75341	0.03477	0.00189	0.04635	2.01050	0.09483	0.01404	0.04742
500	0.75208	0.02678	0.00114	0.03570	2.00503	0.07052	0.00809	0.03526
750	0.75229	0.02132	0.00071	0.02842	2.00361	0.05735	0.00510	0.02868
	$k=3$							
	$\lambda=3.5$				$\lambda=4.75$			
Sample size n	MLE	$\lvert Bias \rvert$	MSE	MRE	MLE	$\lvert Bias \rvert$	MSE	MRE
50	3.53760	0.23908	0.09295	0.06831	4.78091	0.32956	0.17358	0.06938
75	3.52134	0.18366	0.05525	0.05247	4.75693	0.25325	0.10046	0.05332
300	3.49788	0.09134	0.01318	0.02610	4.75270	0.12927	0.02575	0.02721
500	3.49956	0.07265	0.00830	0.02076	4.75288	0.10219	0.01624	0.02151
750	3.49801	0.05799	0.00536	0.01657	4.75367	0.08002	0.01001	0.01685
	$k=5$							
	$\lambda=5.25$				$\lambda=6$			
Sample size n	MLE	$\lvert Bias \rvert$	MSE	MRE	MLE	$\lvert Bias \rvert$	MSE	MRE
50	5.27793	0.26685	0.11264	0.05086	6.02896	0.30118	0.14461	0.05020
75	5.25691	0.21853	0.07441	0.04163	6.00215	0.24376	0.09248	0.04063
300	5.25302	0.10643	0.01804	0.02027	6.00073	0.12317	0.02388	0.02053
500	5.25798	0.08382	0.01120	0.01596	6.01385	0.09423	0.01373	0.01571
750	5.25043	0.07152	0.00792	0.01362	5.99925	0.08025	0.00996	0.01338

Based on the findings from the simulated results displayed in Table 1, it is apparent that the average bias, mean square error (MSE), and mean relative error (MRE) values of the estimator tend to converge towards zero as the sample size increases. Consequently, the estimator for the SSCEr distribution demonstrates precision, accuracy, and stability, thereby establishing its consistency.

5. APPLICATION

To show the usefulness of proposed model, we consider real data set obtained from a glaucoma clinic at the University of Malaya Medical centre, Malaysia. This data consists of the images of the posterior segment of the eyes of 23 patients. Recently, Iftikhar et al. (2022), Maruthan et al. (2022), Rambli et al. (2019), and Ali (2017) used this data to check the applicability of their models.

We compare the performance of stereographic semicircular Erlang distribution with performance of SSCEx. (Phani et al. (2013)), hc-BurrIII, hc-GIW, hc-log logistic, and hc-gamma (Ali 2017) distributions using the Kolmogorov-Smirnov(KS) statistic, the Akaike information criterion(AIC), and Bayesian information criterion(BIC) to find out the best-fitting distribution. All the required statistics are computed and present in Table2 and 3.

Data set:

1.60	1.21	1.46	2.10	1.40	1.82	1.57	1.56	1.85	0.60	1.70	1.97	1.47
	1.74	1.67	1.38	0.53	1.69	1.63	1.56	1.81	2.09	2.29		

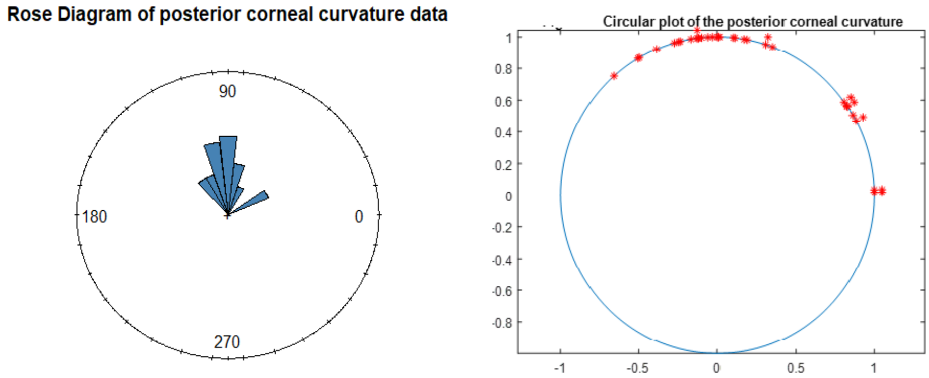

Figure 4. Rose diagram of eye data set (left) and Circular plot of eye data set (right)

Table 2. MLEs and their standard errors for eye data set

Model	$\theta_1(S.E)$	$\theta_2(S.E)$	$\theta_3(S.E)$
SSCEr	5.42656(**0.46194**)	5	-
SSCEx	0.90449(0.18859)	-	-
hc-BurrIII	1.00047(0.22975)	4.28673(0.85739)	-
hc-GIW	0.31027(0.10242)	1.70011(0.23619)	4.80355(2.0366)
hc-log logistic	1.06425(0.08438)	4.38493(0.80058)	-
hc-gamma	5.72203(1.63874)	0.19321(0.05783)	-

Table 3. Summary of statistics

Model	LL	AIC	BIC	KS(p-value)
SSCEr	**-11.10115**	**24.2023**	**25.3370**	**0.1630(0.5741)**
SSCEx	-22.91177	47.8235	48.9590	0.3955(0.0015)
hc-BurrIII	-11.37239	26.7449	29.0158	0.1839(0.4180)
hc-GIW	-18.77315	43.5463	46.9528	0.2719(0.0666)
hc-log logistic	-11.07212	26.1443	28.4152	0.1165(0.9136)
hc-gamma	-11.08750	26.1746	28.4456	0.1698(0.5208)

The higher value of the log-likelihood statistic, along with the smaller values of AIC and BIC, unequivocally indicates that the stereographic semicircular Erlang distribution provides a better fit to the dataset compare to the other competent distribution.

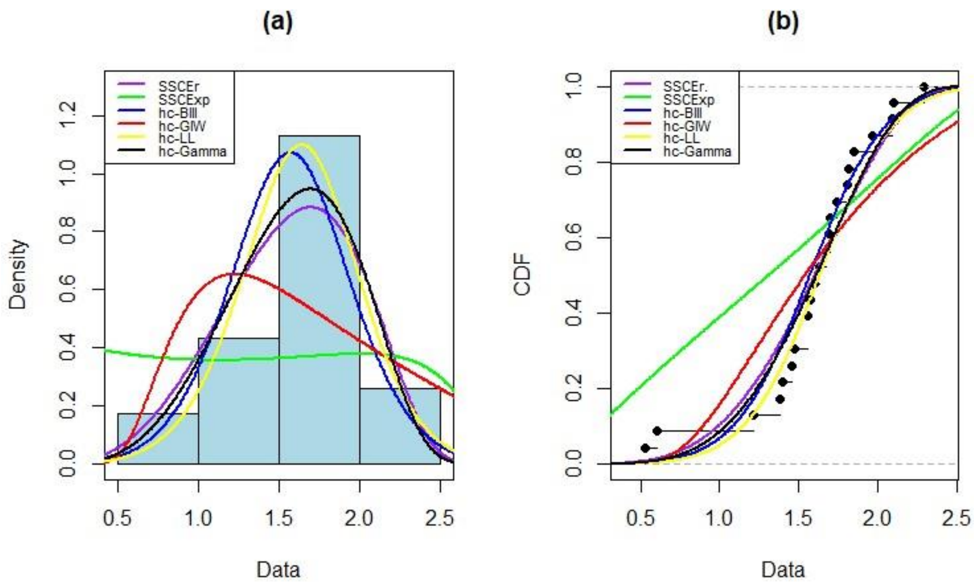

Figure 5. *(*a) Fitted densities of the SSCEr. SSCEx. Hc -Burr III, hc-GIW, hc-log logistic, and hc-gamma models to eye data set. (b) Fitted distribution function of the SSCEr., SSCEx., hc-BurrIII, hc-GIW, hc-log logistic, and hc-gamma models to eye data set

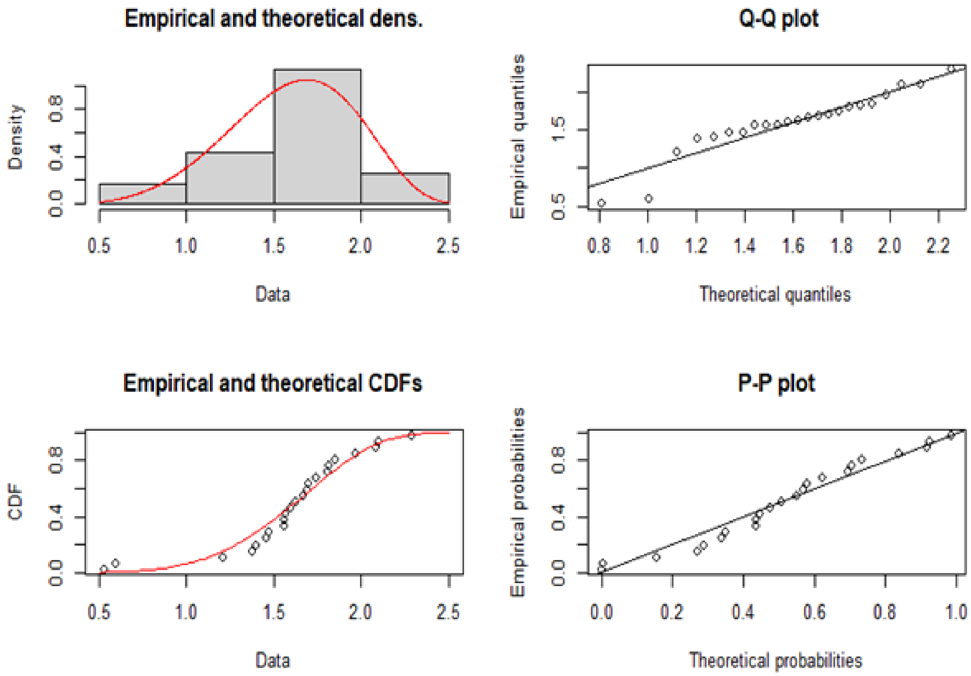

Figure 6. The empirical pdf (top left panel), cdf (bottom left panel), Q-Q (top right panel), and P-P (bottom right panel) plots for eye data set

All the computation is evaluated by using FitdistrPlus, Adequacy Model (Pedro Rafael et al. 2019)

6. CONCLUSION

In this research paper, we introduce the stereographic semicircular Erlang distribution, a novel two-parameter distribution created via the inverse stereographic projection (ISP) technique, building upon the existing stereographic semicircular exponential distribution. We meticulously explore its mathematical properties, conduct simulations to estimate parameters, and empirically analyze data from posterior corneal curvature measurements of 23 patients' eyes. This empirical investigation serves to assess the distribution's adaptability and practical utility. Our findings emphasize the distribution's promise in statistical representation, highlighting its potential for diverse applications and contributing significantly to advancing statistical modeling methodologies.

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