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SOLVING INTUITIONISTIC FUZZY TRANSPORTATION PROBLEM USING GM-R METHOD

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A B S T R A C T

In recent years, researchers have focused on the Intuitionistic Fuzzy Transportation Problem (IFTP) due to its significant and indispensable role in the Transportation Problem (TP). An endeavour is performed in this paper to establish a new technique. This technique works with IFTP based on "Geometric Mean-Ranking(GM-R)"method. To provide a comprehensive understanding of the proposed method, we present and solve extensively discussed numerical examples. The acquired outcomes are subsequently subjected to numerical comparison with those of established methods, confirming the effectiveness of the proposed technique. This study validates the flexibility and simplicity of applying the method to real-life IFTPs for decision-makers.

1. INTRODUCTION

In the global context, delivering services such as goods and value-based services from available sources to diverse destinations like individuals, organizations, shops, and supermarkets presents numerous challenges. Achieving this in a profitable manner requires optimization. The TP has proven to be an effective technique in providing optimal solutions for transporting resources from available sources to destinations in need. Traditional TP, however, is designed to handle precise, crisp data.

Upon careful examination of practical TP data, it becomes evident that the data lacks precision; in other words, it contains vagueness or impreciseness, representing a form of uncertainty. Zadeh's Fuzzy Set (FS), introduced in 1965, has proven effective in handling vague or imprecise data. In response to this, the Fuzzy Transportation Problem (FTP) was developed to address TP within a Fuzzy Environment (FE).

Intuitionistic Fuzzy Set (IFS), emerged as extension of FS, has been found to be particularly suitable for analyzing and organizing uncertainty in certain practical problems compared to traditional FS. Consequently, IFTP has emerged as an extension of FTP, leveraging the enhanced capabilities of IFS. The point that Intuitionistic Fuzzy Number (IFN) belongs to the class of IFS, is remarkable.

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Hitchcock (1941) pioneered the design of the fundamental TP. Subsequently, Bellman and Zadeh (1970) constructed a platform for understanding how decision is made in a FE. Zimmermann (1983) explained about how to raise the quality in solutions through the amalgamation of Linear Programming Problem (LPP) with fuzzy logic. Hussain and Kumar (2012) produced a method to crack the TP involving supplies and demands represented as Triangular Intuitionistic Fuzzy Numbers (TIFN). They utilized the intuitionistic fuzzy Vogel's approximation method and the Intuitionistic Fuzzy Modified Distribution Method (IFMODI). Gani and Abbas (2013) discussed a solution approach for the IFTP through a ranking function, with supplies and demands represented as TIFNs. Pramila and Uthra (2014) investigated a method employing an accuracy function to explore the best solution for IFTP with cost, supply, and demand as TIFNs. Kumar and Hussain (2015) introduced a strategy for handling IFTP involving TIFNs in a single stage. Aggarwal and Gupta (2016) presented a method for solving IFTP through a proposed signed distance ranking method. Ebrahimnejad and Verdegay (2017) described a method aiming to solve IFTP by reducing it to a crisp Linear Programming Problem (LPP) and then applying LPP algorithms. Uthra et al (2017) illustrated a method for providing an optimal solution for IFTP with generalized Trapezoidal Intuitionistic Fuzzy Numbers (TrIFN), using a ranking function to reduce the entire problem to a crisp form. Narayanamoorty and Deepa (2017) introduced the intuitionistic fuzzy Russell's method to solve IFTP. Kumar (2018) tabled a method to explore a solution directly from IFTP, where costs are expressed as TIFNs and supply and demand are presented by crisp numbers. Purushothkumar et al (2018) put forward a technique to solve IFTP utilizing the diagonal optimal algorithm. Hunwisai et al (2019) introduced a method to solve IFTP using the northwest corner rule and the MODI method. Taghaodi (2019) presented a method which optimizes initially obtained feasible solution with the MODI method. Nishad and Abhishekh (2020) illustrated a method with a distance minimizer concept for solving IFTP. Abirami et al (2020) introduced a ranking method to solve IFTP based on signed distance. Rani et al (2023) endeavored to surface a method applying fuzzy branch and bound techniques and IFMODI method to obtain a solution for IFTP. Beg et al (2023) placed afore a technique to investigate a solution for generalized IFTPs using an optimism-based expected value approach.

Numerous currently available methods entail protracted procedures and intricate calculations. Certain approaches specifically address either TrIFNs or TIFNs. Others are tailored for problems wherein the supply and demand are IFNs. Some methods are exclusively designed for scenarios where the cost matrix involves IFNs. Additionally, there are methods dedicated to challenges where supply, demand, and cost are all represented as IFNs. In light of these constructive insights, a novel method is introduced for solving IFTP. This strategy is founded on the "GM-R" approach, offering the flexibility to handle TrIFNs and TIFNs, as well as their generalized counterparts.

This paper is neatly showcased as follows: Second portion provides essential fundamentals concerned to IFNs. Third section is dedicated to a clear illustration of the proposed method, supported by relevant data and examples. In the fourth section, discussions and useful comparisons with other author works are presented. The paper is concluded in the fifth section, summarizing key points.

2. PRELIMINARIES

Definition 2.1:
A FS $\tilde{A}$, where X means universal set and $\mu_{\tilde{A}}(x)$ means Membership Value (MV) function, is defined by $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)), x \in X\}$ and $\mu_{\tilde{A}}(x): X \to [0,1]$.

Definition 2.2:
An IFS $\tilde{A}^I$ in X is a set of the form $\tilde{A}^I = \{(x, \mu_{\tilde{A}^I}(x), \vartheta_{\tilde{A}^I}(x)) : x \in X\}$ where the function $\mu_{\tilde{A}^I}(x): X \to [0,1]$ and $\vartheta_{\tilde{A}^I}(x): X \to [0,1]$ describe the degree of MV and Non-Membership Value (N-MV) of the element $x \in X$ respectively and for every $x \in X$ in $\tilde{A}^I$, $0 \le \mu_{\tilde{A}^I}(x) + \vartheta_{\tilde{A}^I}(x) \le 1$ holds.

$\tilde{A}^I$ is said to be an intuitionistic fuzzy normal if $\exists$ two points $x_0, x_1 \in X$ such that $\mu_{\tilde{A}^I}(x_0) = 1, \vartheta_{\tilde{A}^I}(x_1) = 1$

$\tilde{A}^I$ is said to be an IFN if it is
a) Intuitionistic fuzzy normal
b) $\mu_{\tilde{A}^I}(x)$ is convex. i.e.,
$\mu_{\tilde{A}^I}(x)(\lambda x_1 + (1-\lambda)x_2) \ge min(\mu_{\tilde{A}^I}(x_1), \mu_{\tilde{A}^I}(x_2))$ for every $x_1, x_2 \in X, \lambda \in [0,1]$
c) $v_{\tilde{A}^I}(x)$ is concave. i.e.,
$\vartheta_{\tilde{A}^I}(x)(\lambda x_1 + (1-\lambda)x_2) \le max(\vartheta_{\tilde{A}^I}(x_1), \vartheta_{\tilde{A}^I}(x_2))$ for every $x_1, x_2 \in X, \lambda \in [0,1]$

Definition 2.3:
A generalized TIFN with parameters $q_1 \le p_1 \le q_2 (\le, \ge) p_2 \le p_3 \le q_3$ is denoted as

$$\tilde{A}^I = ((p_1, p_2, p_3; w), (q_1, q_2, q_3; v))$$

Where MV is given by

$$\mu_{\tilde{A}^I}(x) = \begin{cases} w\dfrac{x-p_1}{p_2-p_1}, p_1 < x < p_2 \\ w\dfrac{p_3-x}{p_3-p_2}, p_2 < x < p_3 \\ 0, \quad otherwise \end{cases}$$

N-MV is given by

$$\vartheta_{\tilde{A}^I}(x) = \begin{cases} \dfrac{q_2 - x + v(x-q_1)}{q_2-q_1}, & q_1 < x < q_2 \\ \dfrac{x - q_2 + v(q_3 - x)}{q_3 - q_2}, & q_2 < x < q_3 \\ 1, & otherwise \end{cases}$$

and w is the maximum MV and v is the minimum

N-MV
Such that $\mu_{\tilde{A}^I}(x) \le w \; and \; \vartheta_{\tilde{A}^I}(x) \ge v \; for$ all x,
$0 \le w \le 1, 0 \le v \le 1 \; and \; 0 \le w + v \le 1$
If $w = 1$ and $v = 0$ in generalized TIFN, then it is called TIFN and it is denoted as

$$\tilde{A}^I = \big((p_1, p_2, p_3), (q_1, q_2, q_3)\big)$$

Definition 2.4:
A generalized TrIFN with parameters $q_1 \le p_1 \le q_2 \le p_2 \le p_3 \le q_3 \le p_4 \le q_4$ is denoted as

$$\tilde{A}^I = \big((p_1, p_2, p_3, p_4; w), (q_1, q_2, q_3, q_4; v)\big)$$

Where MV is given by

$$\mu_{\tilde{A}^I}(x) = \begin{cases} w\dfrac{x - p_1}{p_2 - p_1}, p_1 < x < p_2 \\ w, \quad p_2 < x < p_3 \\ w\dfrac{p_4 - x}{p_4 - p_3}, p_3 < x < p_4 \\ 0, \quad otherwise \end{cases}$$

N-MV is given by

$$\vartheta_{\tilde{A}^I}(x) = \begin{cases} \dfrac{q_2 - x + v(x - q_1)}{q_2 - q_1}, \quad q_1 < x < q_2 \\ v, \quad q_2 < x < q_3 \\ \dfrac{x - q_3 + v(q_4 - x)}{q_4 - q_3}, \quad q_3 < x < q_4 \\ 1, \quad otherwise \end{cases}$$

and w is the maximum MV and v is the minimum N-MV
Such that $\mu_{\tilde{A}^I}(x) \le w and \vartheta_{\tilde{A}^I}(x) \ge v for$ all x
$0 \le w \le 1, 0 \le v \le 1 \; and \; 0 \le w + v \le 1$

In the above definition, if we let $b_2 = b_3$ (and hence $a_2 = a_3$), then generalized TrIFN becomes generalized TIFN. If $w = 1$ and $v = 0$ in the definition of generalised TrIFN, then it is called TrIFN and it is denoted as

$$\tilde{A}^I = \big((p_1, p_2, p_3, p_4), (q_1, q_2, q_3, q_4)\big)$$

3. PROPOSED METHOD

3.1 Resultant membership function

Let $\mu_{\tilde{A}^I}(x)$ and $\nu_{\tilde{A}^I}(x)$ be MV and N-MV functions of an IFN $\tilde{A}^I$, then the resultant membership function $R_{\tilde{A}^I}(x)$ is defined to be

$$R_{\tilde{A}^I}(x) = \mu_{\tilde{A}^I}(x) - \nu_{\tilde{A}^I}(x)$$

3.2 Support

The support of an IFN $\tilde{A}^I$ is $S(\tilde{A}^I) = \{x : R_{\tilde{A}^I}(x) \ge 0\}$

3.3 Proposition

If $\tilde{A}^I = \big((p_1, p_2, p_3, p_4; w), (q_1, q_2, q_3, q_4; v)\big)$ be a TrIFN then the support of $\tilde{A}^I$ is
$S(\tilde{A}^I) = [MP_1, MP_2]$where
$MP_1 = \frac{wp_1(q_2-q_1)+(p_2-p_1)(q_2-vq_1)}{w(q_2-q_1)+(1-v)(p_2-p_1)}$ and
$MP_2 = \frac{wp_4(q_4-q_3)+(p_4-p_3)(q_3-vq_4)}{w(q_4-q_3)+(1-v)(p_4-p_3)}$.

3.4 Proposition

If $\tilde{A}^I = \big((p_1, p_2, p_3; w), (q_1, q_2, q_3; v)\big)$ be a TIFN, then the support of $\tilde{A}^I$is $S(\tilde{A}^I) = [MP_1, MP_2]$where
$MP_1 = \frac{p_1 w(q_2-q_1)+(p_2-p_1)(q_2-vq_1)}{w(q_2-q_1)+(1-v)(p_2-p_1)}$ and
$MP_2 = \frac{wp_3(q_3-q_2)+(p_3-p_2)(q_2-vq_3)}{w(q_3-q_2)+(1-v)(p_3-p_2)}$.

3.5 Geometric Mean-Ranking (GM-R) Method

If $\tilde{A}^I$ be an IFN with $\mu_{\tilde{A}^I}(x)$ and $\nu_{\tilde{A}^I}(x)$ be MV and N-MV functions respectively and $S(\tilde{A}^I)$ be the support of $\tilde{A}^I$. Then the crisp value concerned with $\tilde{A}^I$, denoted as $G_{\tilde{A}^I}$, is defined to be

$$G_{\tilde{A}^I} = Exp\left[\frac{\int_{S(\tilde{A}^I)} R_{\tilde{A}^I}(x).\ln x dx}{\int_{S(\tilde{A}^I)} R_{\tilde{A}^I}(x) dx}\right] \quad (1)$$

Where $R_{\tilde{A}^I}(x)$ is resultant membership function of $\tilde{A}^I$.

3.6 Ranking procedure

Let $\tilde{A}^I$ and $\tilde{B}^I$ be two IFNs.
Step1: Calculate the Supports $S(\tilde{A}^I)$ and $S(\tilde{B}^I)$ of given $\tilde{A}^I$ and $\tilde{B}^I$ respectively by either proposition 3.3 or 3.4
Step2: Calculate crisp values of $\tilde{A}^I$and $\tilde{B}^I$ using the formula (1) and denote them as $G_{\tilde{A}^I}$ and $G_{\tilde{B}^I}$ respectively.
Step3: The raking order can be identified by following cases
If $G_{\tilde{A}^I} < G_{\tilde{B}^I}$ then $\tilde{A}^I \prec \tilde{B}^I$.
If $G_{\tilde{A}^I} > G_{\tilde{B}^I}$ then $\tilde{A}^I \succ \tilde{B}^I$.
If $G_{\tilde{A}^I} = G_{\tilde{B}^I}$ then $\tilde{A}^I \sim \tilde{B}^I$

3.7 Intuitionistic Fuzzy Transportation Problem (IFTP)

If any one of the quantities in TP is IFN, then it is seen as IFTP. In general, IFTP is of the following form

Minimize $\tilde{Z}^I = \sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{c}_{ij}^I \tilde{x}_{ij}^I$
Subject to $\sum_{j=1}^{n} \tilde{x}_{ij}^I = \tilde{a}_i^I, \quad i = 1,2,3 \dots \dots m,$
$\sum_{i=1}^{m} \tilde{x}_{ij}^I = \tilde{b}_i^I, \quad j = 1,2,3 \dots \dots . n$ and $\tilde{x}_{ij}^I \ge 0$ for all $i and j$

	1	2	...	n	$\tilde{a}_i^I$
1	$\tilde{c}_{11}^I$	$\tilde{c}_{12}^I$	...	$\tilde{c}_{1n}^I$	$\tilde{a}_1^I$
2	$\tilde{c}_{21}^I$	$\tilde{c}_{22}^I$	...	$\tilde{c}_{2n}^I$	$\tilde{a}_2^I$
⋮	⋮	⋮	⋮	⋮	⋮
M	$\tilde{c}_{m1}^I$	$\tilde{c}_{m2}^I$	...	$\tilde{c}_{mn}^I$	$\tilde{a}_m^I$
$\tilde{b}_j^I$	$\tilde{b}_1^I$	$\tilde{b}_2^I$	...	$\tilde{b}_n^I$	$\sum_{i=1}^{m} \tilde{a}_i^I = \sum_{j=1}^{n} \tilde{b}_j^I$

where,

$\tilde{c}^I_{ij}$ is the fuzzy cost of transportation of one unit of the goods from i^{th} source to the j^{th} destination.
$\tilde{x}^I_{ij}$ is the quantity transportation from i^{th} source to the j^{th} destination.
$\tilde{a}^I_i$ is the total availability of the goods at i^{th} source.
$\tilde{b}^I_i$ is the total demand of the goods at j^{th} destination.
$\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}^I_{ij}\,\tilde{x}^I_{ij}$ is total fuzzy transportation cost.
If $\sum_{i=1}^{m}\tilde{a}^I_i = \sum_{j=1}^{n}\tilde{b}^I_i$, then IFTP is said to be balanced.
If $\sum_{i=1}^{m}\tilde{a}^I_i \neq \sum_{j=1}^{n}\tilde{b}^I_i$, then IFTP is said to be unbalanced.

There are also different cases in IFTP like
Case1. Only the cost coefficients are IFNs and the supplies and demands are crisp.
Case2. The IFTP in which all the quantities are IFNs, then it is treated as fully IFTP.

3.8 Algorithm

The algorithm contains following steps.
Step1: Construct an IFTP from the given data and get it represented in tabular form.
Step2: Applying the GM-R method, the IFTP at step1 is reduced to a crisp TP.
Step3: At this stage, the problem with crisp values of supply, demand and cost, is formulated as a conventional LPP as following
Minimize $Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$
Subject to $\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1,2,3 \ldots\ldots m,$
$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1,2,3 \ldots\ldots n$ and $x_{ij} \geq 0$
for all $i and j$
Step4: In this step, the LPP obtained in Step3, is solved and optimal solution is obtained.
Step5: Minimum Transportation Cost (MTC) is given in the crisp form using optimal solution obtained in step4 and crisp cost matrix.

4. NUMERICAL COMPARISONS AND DISCUSSION

The perusal of numerical comparisons, which contributes significancy for the proposed IFTP method, is conducted here. This perusal is done with mostly observed significant works of others. From this perusal, the worthy points of the proposed IFTP are identified.

Example 1:

Considered IFTP from Kumar (2018) with three sources S1, S2, S3 and three destinations D1, D2, D3. The transportation cost is given in TIFNs. And supply and demand are given in crisp values. The problem is shown in Table1. Find the optimal value.

Step1:

Table 1. Step1 for Example 1.

$Source$	$D1$	$D2$	$D3$	$Supply\ (a_i)$
$S1$	$\tilde{c}^I_{11}$	$\tilde{c}^I_{12}$	$\tilde{c}^I_{13}$	18
$S2$	$\tilde{c}^I_{21}$	$\tilde{c}^I_{22}$	$\tilde{c}^I_{23}$	12
$S3$	$\tilde{c}^I_{31}$	$\tilde{c}^I_{32}$	$\tilde{c}^I_{33}$	4
$Demannd\ (b_i)$	8	16	10	

where, $\tilde{c}^I_{11} = ((7,21,29),(2,21,34))$,
$\tilde{c}^I_{12} = ((7,20,57),(3,20,61))$,
$\tilde{c}^I_{13} = ((12,25,56),(8,25,60))$,
$\tilde{c}^I_{21} = ((8,9,16),(2,9,22))$,
$\tilde{c}^I_{22} = ((4,12,35),(1,12,38))$,
$\tilde{c}^I_{23} = ((6,14,28),(3,14,31))$,
$\tilde{c}^I_{31} = ((5,9,22),(2,9,25))$,
$\tilde{c}^I_{32} = ((10,15,20),(5,15,25))$,
$\tilde{c}^I_{33} = ((6,14,28),(3,14,31))$.

Step2:

Table 2. Step2 for Example1.

$Source$	$D1$	$D2$	$D3$	$Supply(a_i)$
$S1$	19.7768	23.3704	27.588	18
$S2$	10.1578	14.0952	14.8007	12
$S3$	10.3456	14.9378	14.319	4
$Demand(b_j)$	8	16	10	

Step3:
Corresponding to a crisp TP obtained at step2, the crisp LPP is formulated as following

Minimize:
$$Z = 19.7768x_{11} + 23.3704x_{12} + 27.588x_{13} + 10.1578x_{21} + 14.0952x_{22} + 14.8007x_{23} + 10.3456x_{31} + 14.9378x_{32} + 14.319x_{33}$$
Subject to
$$19.7768x_{11} + 23.3704x_{12} + 27.588x_{13} = 18,$$
$$10.1578x_{21} + 14.0952x_{22} + 14.8007x_{23} = 12,$$
$$10.3456x_{31} + 14.9378x_{32} + 14.319x_{33} = 4,$$
$$19.7768x_{11} + 10.1578x_{21} + 10.3456x_{31} = 8,$$
$$23.3704x_{12} + 14.0952x_{22} + 14.9378x_{32} = 16,$$
$$27.588x_{13} + 14.8007x_{23} + 14.319x_{33} = 10,$$
$$x_{ij} \geq 0, \quad i = 1,2,3, and j = 1,2,3.$$
Step4:

Table 3. Step4 for Example1.

$Source$	$D1$	$D2$	$D3$	$Supply(ai)$
$S1$	2	16	0	18
$S2$	6	0	6	12
$S3$	0	0	4	4
$Demand(bi)$	8	16	10	

Hence optimal solution is
$$x_{11} = 2, x_{12} = 16, x_{21} = 6, x_{23} = 6, x_{33} = 4$$

Step5:
The MTC is

$$2 \times 19.7768 + 16 \times 23.3704 + 6 \times 10.1578 + 6 \times 14.8007 + 4 \times 14.319 = 620.507$$

Thus, for the given IFTP, the optimal solution and optimum value in crisp form by the proposed method are $x_{11} = 2, x_{12} = 16, x_{21} = 6, x_{23} = 6, x_{33} = 4$ and$Z_{propsedmethod} = 620.507$ respectively.

Kumar (2018) applied LPP method to solve IFTP in which IFTP is converted to a conventional LPP by the use of ranking function. And then obtained LPP is solved.The optimal solution by Kumar (2018) is

$$x_{11} = 8, x_{12} = 10, x_{22} = 6, x_{23} = 6, x_{33} = 4$$

Minimum Intuitionistic Fuzzy Transportation Cost(MIFTC) by Kumar (2018)is $\big((202,588,1256),(74,588,1384)\big)$ and crisp optimum value provided by Kumar (2018) is 682

$$MTC_{proposedmethod} = 620.507 < MTC_{\text{Kumar}} = 682$$

The proposed method is noticed to give superior solution with less transportation cost.

In the above Example1, cost parameters are provided by TIFNs. And supply and demand are expressed by crisp numbers.

Example 2:

Considered IFTP from Kumar (2018) with four sources S1, S2, S3, S4. and four destinations D1, D2, D3, D4. The transportation cost is given in TIFNs and supply and demand are given in crisp values. The problem is shown in Table 4. Find the optimal value.

Step1:

Table 4. Step1 for Example 2.

$Source$	$D1$	$D2$	$D3$	$D4$	$Supply$ (a_i)
$S1$	$\tilde{c}_{11}^I$	$\tilde{c}_{12}^I$	$\tilde{c}_{13}^I$	$\tilde{c}_{14}^I$	4
$S2$	$\tilde{c}_{21}^I$	$\tilde{c}_{22}^I$	$\tilde{c}_{23}^I$	$\tilde{c}_{24}^I$	6
$S3$	$\tilde{c}_{31}^I$	$\tilde{c}_{32}^I$	$\tilde{c}_{33}^I$	$\tilde{c}_{34}^I$	8
$S4$	$\tilde{c}_{41}^I$	$\tilde{c}_{42}^I$	$\tilde{c}_{43}^I$	$\tilde{c}_{44}^I$	10
$Demand$ (b_j)	4	5	12	7	

where, $\tilde{c}_{11}^I = \big((27,50,109),(30,50,123)\big)$,
$\tilde{c}_{12}^I = \big((56,67,111),(40,67,127)\big)$,
$\tilde{c}_{13}^I = \big((8,22,120),(4,22,124)\big)$,
$\tilde{c}_{14}^I = \big((75,100,128),(62,100,141)\big)$,
$\tilde{c}_{21}^I = \big((52,68,93),(44,68,101)\big)$,
$\tilde{c}_{22}^I = \big((43,90,119),(35,90,127)\big)$,
$\tilde{c}_{23}^I = \big((34,56,93),(18,56,109)\big)$,
$\tilde{c}_{24}^I = \big((60,70,89),(50,70,99)\big)$,
$\tilde{c}_{31}^I = \big((72,80,109),(58,80,123)\big)$,
$\tilde{c}_{32}^I = \big((10,15,20),(5,15,25)\big)$,
$\tilde{c}_{33}^I = \big((4,6,19),(1,16,22)\big)$,
$\tilde{c}_{34}^I = \big((75,100,128),(62,100,141)\big)$,
$\tilde{c}_{41}^I = \big((23,40,81),(17,40,87)\big)$,
$\tilde{c}_{42}^I = \big((44,58,90),(38,58,96)\big)$,
$\tilde{c}_{43}^I = \big((63,89,109),(49,89,123)\big)$,
$\tilde{c}_{44}^I = \big((64,72,95),(51,72,108)\big)$.

Step2:

Table 5. Step2 for Example 2.

$Source$	$D1$	$D2$	$D3$	$D4$	$Supply(a_i$
$S1$	55.305	72.518	33.814	100.31	4
$S2$	69.366	86.540	58.159	71.506	6
$S3$	83.675	91.005	104.35	69.369	8
$S4$	43.550	60.814	87.7521	74.632	10
$Demand(b$	4	5	12	7	

Step3:
Corresponding to crisp TP obtained at step2, the crisp LPP is formulated as following

Minimize

$$Z = 55.3052x_{11} + 72.5186x_{12} + 33.8146x_{13} + 100.31x_{14} + 69.3669x_{21} + 86.5405x_{22} + 58.159 + 71.5067x_{24} + 83.675x_{31} + 91.0057x_{32} + 104.35x_{33} + 69.3699x_{34} + 43.5503x_{41} + 60.8149x_{42} + 87.7521x_{43} + 74.6323x_{44}$$

Subject to

$$55.305x_{11} + 72.52x_{12} + 33.814x_{13} + 100.31x_{14} = 4$$
$$69.366x_{21} + 86.541x_{22} + 58.159x_{23} + 71.51x_{24} = 6$$
$$83.675x_{31} + 91.01x_{32} + 104.35x_{33} + 69.369\,x_{34} = 8$$
$$43.55\,x_{41} + 60.814\,x_{42} + 87.75\,x_{43} + 74.63x_{44} = 10$$
$$55.31x_{11} + 69.366x_{21} + 83.675x_{31} + 43.55\,x_{41} = 4$$
$$72.519x_{12} + 86.51x_{22} + 91.01x_{32} + 60.815\,x_{42} = 6$$
$$33.815x_{13} + 58.159x_{23} + 104.35x_{33} + 87.75\,x_{43} = 8$$
$$100.31x_{14} + 71.51\,x_{24} + 69.37\,x_{34} + 74.63\,x_{44} = 10$$
$$x_{ij} \geq 0,\ \ i = 1,2,3\ andj = 1,2,3,4$$

Step4:

Table 6. Step4 for Example 2.

$Source$	$D1$	$D2$	$D3$	$D4$	$Supply(a_i)$
$S1$	0	0	4	0	4
$S2$	0	0	6	0	6
$S3$	0	0	2	6	8
$S4$	4	5	0	1	10
$Demand(b_j)$	4	5	12	7	

Hence optimal solution is $x_{13} = 4, x_{23} = 6, x_{33} = 2, x_{34} = 6, x_{41} = 4, x_{42} = 5, x_{44} = 1$

Step5:
The MTC is

$$4 \times 33.8146 + 6 \times 58.159 + 2 \times 104.35 + 6 \times 69.3699 + 4 \times 43.5503 + 5 \times 60.8149 + 1 \times 74.6323 = 1662.04$$

Thus, for the given IFTP, the optimal solution and optimum value in crisp form by the proposed method are $x_{13} = 4, x_{23} = 6, x_{33} = 2, x_{34} = 6, x_{41} = 4,$ $x_{42} = 5, x_{44} = 1$and $Z_{propsedmethod} = 1662.04$ respectively.The optimal solution by the authors Kumar (2018)is $x_{13} = 4, x_{23} = 6, x_{33} = 1, x_{34} = 7, x_{41} = 4, x_{42} = 5, x_{44} = 1$. MIFTC by the author Kumar (2018) is $\big((1060, 1537, 2722), (815,1537,2967)\big)$
Crisp value of MIFTC is 1773.

$$MTC_{proposedmethod} = 1662.04 < MTC_{\text{Kumar}} = 1773$$

The proposed method is noticed to give better solution with less transportation cost.
In the above Example2, cost parameters are provided by TIFNs,and supply and demand are expressed by crisp numbers. In the next example3, all the parameters are expressed by TrIFNs.

Example 3:

Considered IFTP from Ebrahimnejad and Verdegay (2017) with two sources S1 and S2 and three destinations D1, D2, D3. The transportation cost with supply and demand is given in TrIFNS. And problem is shown in Table 7. Find out the optimal value.
Step1:

Table 7. Step1 for Example 3.

$Source$	$D1$	$D2$	$D3$	$Supply$ $(\tilde{a}_i^I)$
$S1$	$\tilde{c}_{11}^I$	$\tilde{c}_{12}^I$	$\tilde{c}_{13}^I$	$\tilde{a}_1^I$
$S2$	$\tilde{c}_{21}^I$	$\tilde{c}_{22}^I$	$\tilde{c}_{23}^I$	$\tilde{a}_2^I$
$Demand\ (\tilde{b}_i^I)$	$\tilde{b}_1^I$	$\tilde{b}_2^I$	$\tilde{b}_3^I$	

where,
$\tilde{c}_{11}^I = \big((10,20,30,40), (5,15,35,45)\big)$
$\tilde{c}_{12}^I = \big((50,60,70,90), (45,55,75,95)\big)$
$\tilde{c}_{13}^I = \big((80,90,110,120), (75,85,115,125)\big)$
$\tilde{c}_{21}^I = \big((60,70,80,90), (55,65,85,95)\big)$
$\tilde{c}_{22}^I = \big((70,80,100,120), (65,75,115,125)\big)$
$\tilde{c}_{23}^I = \big((20,30,50,60), (15,25,35,65)\big)$
$\tilde{a}_1^I = \big((60,80,100,120), (50,70,110,130)\big)$
$\tilde{a}_2^I = \big((40,60,80,100), (30,50,90,110)\big)$
$\tilde{b}_1^I = \big((30,50,70,90), (20,40,80,100)\big)$
$\tilde{b}_2^I = \big((20,30,40,50), (15,25,45,55)\big)$
$\tilde{b}_3^I = \big((50,60,70,80), (45,55,75,85)\big)$

Step2:

Table 8. Step2 for Example 3.

$Source$	$D1$	$D2$	$D3$	$Supply(a_i)$
$S1$	24.2625	64.7264	94.8134	89
$S2$	74.7633	84.7913	34.4844	69
$Demand(b_j)$	59	34	65	

Step3:
Corresponding to a crisp TP obtained at step2, the crisp LPP is formulated as following

Minimize

$$Z = 24.2625x_{11} + 64.7264x_{12} + 94.8134x_{13} + 74.7633x_{21} + 84.7913x_{22} + 34.4844x_{23}$$

Subject to

$$24.2625x_{11} + 64.7264x_{12} + 94.8134x_{13} = 89,$$
$$74.7633x_{21} + 84.7913x_{22} + 34.4844x_{23} = 69,$$
$$246.49\ x_{11} + 74.7633x_{21} = 59,$$
$$64.7264x_{12} + 84.7913x_{22} = 34,$$
$$94.8134x_{13} + 34.4844x_{23} = 65,$$
$$x_{ij} \geq 0,\ \ i = 1,2,3\ andj = 1,2,3,4$$

Step4:

Table 9. Step 4 for Example 3.

$Source$	$D1$	$D2$	$D3$	$Supply(a_i)$
$S1$	59	30	0	89
$S2$	0	4	65	69
$Demand(b_j)$	59	34	65	

Hence optimal solution is $x_{11} = 59, x_{12} = 30, x_{13} = 0, x_{21} = 0, x_{22} = 4, x_{23} = 65$

Step5:
The MTC is

$$59 \times 24.2625 + 30 \times 64.7264 + 4 \times 84.7913 + 65 \times 34.4844 = 5953.93$$

Accordingly, for the given IFTP, the optimal solution and optimum value in crisp form by the proposed method are $x_{11} = 59, x_{12} = 30, x_{13} = 0, x_{21} = 0, x_{22} = 4, x_{23} = 65$
And $Z_{propsedmethod} = 5953.93$ respectively.
Ebrahimnejad and Verdegay (2017) converted IFTP into a crisp LPP and solved by standard LPP methods.MIFTC by Ebrahimnejad and Verdegay is $\big((3300, 5800, 9100, 13200), (2350, 4450, 11050, 15550)\big)$ and its crisp value by the proposed method is 7530.96

$$MTC_{proposedmethod} = 5953.93 < MTC_{\text{Ebrahimnejad and Verdegay}} = 7530.96$$

The proposed method is remarked to give finer solution with less transportation cost.

5. CONCLUSION

Introducing a novel strategy for Intuitionistic Fuzzy Transportation Problems (IFTP), this approach is grounded in the GM-R method. Specifically designed to address various types of TIFNs and TrIFNs, Generalized TIFNs and TrIFNs, this method furnishes optimal solutions and optimum values in a precise, crisp format—providing a significant advantage for decision-makers and experts. The application of this method is straightforward, requiring minimal complexity in computations. Its validity is scrutinized through a comparative study against methods that have undergone multiple evaluations. Notably, the method demonstrates increased efficiency, evident in superior optimal solutions as observed in numerical comparisons.

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