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PHYSICAL AND MATHEMATICAL MODEL OF THE MECHANISM OF ULTRASONIC DEHYDRATION OF MATERIALS WITHOUT PHASE TRANSITION OF LIQUID INTO VAPOR

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A B S T R A C T

The paper theoretically investigates the possibility of ultrasonic oscillations to remove moisture from capillary-porous materials by dispersing it under the action of shock waves formed during cavitation phenomena in the liquid phase. A physical and mathematical model based on analysis of slow growth of cavitation bubble of distorted cylindrical shape taking into account influence of cylindrical capillary walls limiting its oscillations was developed and presented. The optimum range (150-170 dB) of sound pressure levels (at the oscillation frequency of 22 kHz) at which dispersion of moisture is realized, was found. It was found that the optimal conditions of exposure to ultrasonic oscillations (according to the geometry of the sample) on the dried material are realized when the size or thickness of the layer of the dried material corresponds to the length of the ultrasonic oscillations in the air.

1. INTRODUCTION

A significant amount of all energy produced is used for drying processes in various branches of industrial production. According to the forecasts, the cost of energy resources for drying will only increase.

This determines the relevance and significant interest in research aimed at developing new, low-energy and low-temperature methods of drying various materials having different internal structure.

One of the most promising options for improving the efficiency of traditional methods of drying is drying in high-intensity ultrasonic fields, the interest in which has increased significantly over the past 10 years, both from the scientific community, and from research foundations and sectoral ministries of the United States, Japan, China, South Korea, Romania and Russia.

The relevance and necessity of implementing such drying is evidenced by the experimentally confirmed ability of ultrasonic oscillations not only to accelerate the drying process, but also to reduce losses of vitamins

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(for example, apples, strawberries, papaya, etc.) from 8.7 to 21.2% (Boucher, 1959; Carcel et al., 2017; Legay et al., 2011; Musielak et al., 2016; Tsai et al., 2004; Kishore et al., 2022). In the future, this ability of ultrasonic oscillations preserve the useful properties of dried products has been repeatedly confirmed by foreign scientists (on the example of products of plant origin; for ultrasound + microwave drying; for ultrasound + infrared drying, etc.) (Baslar et al., 2016; Carcel et al., 2017; Onwude et al., 2017; Rodrıguez et al., 2018; Szadzinska et al., 2020) and Russian scientists (on the example of food preparation for polar expeditions) (Beck et al., 2014).

In addition, the use of ultrasonic oscillations allows, given other equal conditions of realization of the drying process, to obtain products with lower residual moisture content. All this makes it possible to consider the ultrasonic drying method as a unique technology that allows obtaining products with long-term storage with quality characteristics unattainable with other methods of drying.

This is explained by the following advantages of the ultrasonic drying method:

1) the ability to significantly increase the coefficient of mass transfer within and reduce the thickness of the boundary layer on the surface of the dried material;
2) the ability to significantly reduce energy costs through moisture removal without transferring it into the gaseous state, by its mechanical removal from the pores of the dried material due to the inverse sound-capillary effect and atomization from the surface in the form of fine aerosol due to the formation of surface capillary waves;
3) low heating of the dried product (removal of moisture at low temperatures, or in without increasing the temperature in principle), due to the intensification of processes of liquid extraction from and evaporation of the liquid phase from the surface of the solid.

However, despite these advantages, the real potential of ultrasonic exposure is still far from full understanding and effective practical use due to the following major problems:

- lack of data on the modes and conditions of formation of the ultrasonic field on the surface, ensuring the most effective course of the drying process;
- implementation of the process in a regime of complete phase transition of moisture removed (which does not allow acceleration of the process by more than 20-30%) without investigation of conditions of ultrasonic influence that ensure removal of moisture without its conversion into vapor due to initiation in the dried material and its surface of non-linear effects such as: cavitation moisture atomization, moisture diffusion under the action of sound-capillary effect, etc.

Therefore, identifying the optimal modes and conditions of the drying process is an essential task.
It is obvious that removal of moisture with complete phase transition turns out to be energetically unprofitable even when reaching the limiting evaporation rate due to increasing the interphase surface and imposition of acoustic flows, which is associated with a high value of the specific heat of vaporization of the moisture mass unit.

Nowadays, many researchers have shown that depending on the level of generated sound pressure the structure of energy consumption in the process of ultrasonic drying changes. Thus, starting from a certain level of sound pressure, there is ultrasonic dispersion, which is several orders of magnitude more energy-efficient than the removal of moisture by translation into steam.

The process of ultrasonic dispersion itself is well studied by many researchers and is clearly associated with the phenomenon of cavitation, in which the slamming bubbles atomize the liquid. However, the application of the theory and mechanisms of ultrasonic cavitation to the drying process has a number of peculiarities. In particular, the dried material, in most cases, does not have moisture on the surface in the form of even a thin film (which evaporates rather quickly by natural convection).

Since free moisture with physical-mechanical bond is located in capillaries and small pores, it may well be removed by dispersion due to cavitation. Respectively it can be considered that diameter of dispersion surface on the side of each capillary is equal to diameter of capillary, i.e. in bodies with large capillaries is about 20 μm.

Therefore to study and analyze the process of moisture removal without phase transition it is proposed to develop a phenomenological model of cavitation and moisture extraction from the capillary of the dried material, to analyze the physical mechanism of processes and theoretically identify modes and conditions of ultrasonic influence that ensure the realization of this mechanism.

2. SUBSTANTIATION OF THE PHYSICAL MECHANISM OF CAVITATION IN POROUS MATERIAL WHEN EXPOSED THROUGH A GAS GAP

According to commonly accepted theories of cavitation in an infinite volume of a continuous liquid phase, the cavitation nucleus, which initially has a spherical shape, expands to a large size (20...100 μm) during the half-period of ultrasonic oscillation rarefaction, and then collapses during the half-period of compression (Nikolyuk, 2016; Bhangu & Ashokkumar, 2016).

To evaluate the possibility of applying the existing theoretical provisions describing the dynamics of the spherical cavitation bubble to capillary-porous materials under the ultrasonic influence through the gas gap the calculation of the oscillation intensity in the liquid for the sound pressure level of 160 dB was carried out. The sound pressure amplitude p_A at 160 dB is as follows:

$$p_A = \sqrt{2} \cdot 20 \cdot 10^{-6} \cdot 10^{\frac{160}{20}} = 2820\ Pa\ .$$

The intensity corresponding to this sound pressure amplitude is

$$I = \frac{p_A^{\ 2}}{2\rho c} = 2,6\ \frac{W}{m^2} = 0,00026\ \frac{W}{cm^2},$$

where ρ is water density, kg/m^3; c is sound velocity in water, m/s.

Thus, according to (Khmelev et al., 2015; Rosenberg, 1968), developed cavitation in liquid occurs at intensities not less than 0.2-0.5 W/cm^2. Thus, it is necessary to investigate alternative mechanisms of cavitation occurrence at much lower sound pressure amplitudes.

Therefore, an attempt has been made to explain the occurrence of cavitation during a very large number of oscillation periods (>1000). For this purpose the mechanism of straightened diffusion is considered as the only one known at the moment (Margulis, 1995). Checking the possibility of growth of the bubble to large sizes due to straightened diffusion has shown that this mechanism is not valid because, according to the equation of straightened diffusion:

$$\frac{dm}{dt} = \frac{\frac{8}{3}\pi D C_0 R_0 \left(\frac{p_m}{p_0}\right)^2}{\left(1-\xi^2(R_0)\right)^2 + \xi^2(R_0) d^2(R_0)} - 4\pi D_r C_0 R_0\ ,$$

where m - mass of gas inside the bubble, kg; D - coefficient of forward diffusion of dissolved gas in liquid, m^2/s; D_r - coefficient of reverse diffusion of dissolved gas in liquid, m^2/s; R_0 - initial radius of the bubble, m; p_m - sound pressure amplitude in liquid, Pa; C_0 - initial concentration of undissolved gas in liquid; ξ - dimensionless frequency:

$$\xi(R_0) = \left(\frac{\omega}{\omega_0(R_0)}\right)^2;\ d(R_0) = \frac{1}{Q(R_0)} = \frac{\omega\eta}{K_c(R_0)},$$

η - viscosity of liquid; p_0 - static pressure in liquid, Pa; ω - circular frequency of acoustic influence, s^{-1}; ω_0 - resonant frequency of cavitation bubble, s^{-1}, determined by Minnaert formula $\omega_0 = \frac{1}{R_0}\sqrt{\frac{3\gamma}{\rho}\left(\frac{2\sigma}{R_0} + p_0\right)}$ (for cavitation nuclei with radius less than 1 µm); K_c - compressibility coefficient of the cavitation bubble, equal to $K_c = -\frac{1}{V}\frac{dV}{dp} = \frac{1}{\gamma p_0}$ and following expression for the bubble radius depending on the time of the bubble to a radius of at least 20 µm (Krasilnikov & Krylov, 1984): $R_0(\tau) = \sqrt[8]{R_0^{\,8}(0) + \frac{384 I D C_0 \sigma^2}{\rho^2 p_0^{\,4} \omega^6 \eta^2}\tau}$, necessary to form a shock wave, sufficient to form droplets, grows in 106 periods of oscillation and more, which also does not explain the phenomenon of dispersion.

Therefore, to identify other mechanisms of possible cavitation, the deviation of the bubble shape from the spherical shape was taken into account, since the bubble is located in the capillary volume bounded by the side walls, and the radius of the capillary is comparable to the bubble size that is minimally necessary for collapse with the formation of a shock wave (Khmelev et al., 2015).

To estimate the shape of cavitation bubble the boundary problem of motion of liquid in the vicinity of cavitation bubble enclosed in capillary walls was formulated. The motion of the liquid volume is described by the Laplace equation with respect to the velocity potential:

$$\Delta\varphi = 0\ .$$

On the surface of the cylindrical capillary the kinematic boundary condition is valid:

$$\frac{\partial\varphi}{\partial r} = \frac{dR_{cyl}}{dt}\ .$$

On the cavitation bubble wall the dynamic condition is as follows

$$p_0 - \rho\frac{\partial\varphi}{\partial t} - \rho\frac{|\nabla\varphi|^2}{2} = p_{b0}\left(\frac{2R_0^{\,3}}{\int_{-1}^{1} R^2(\cos\theta, t)\, d(\cos\theta)}\right)^{\gamma} - 2\sigma k$$

and kinematic condition is as follows

$$\frac{\partial R}{\partial t}(\cos\theta,t)=\left(1+\frac{\frac{\partial R}{\partial\tau}\cos\theta}{R(\cos\theta,t)}\right)\cdot$$

$$\cdot\left(\frac{\partial\varphi}{\partial r}\sin\theta+\frac{\partial\varphi}{\partial z}\cos\theta\right)-\frac{\partial R}{\partial\tau}\frac{\frac{\partial\varphi}{\partial z}}{R(\cos\theta,t)}$$

When a liquid moves in a cylindrical capillary whose walls make oscillations, in the absence of a cavitation bubble, the velocity potential is determined according to the following expression:

$$\varphi_{no\,bub}=\frac{1}{R_{cyl}}\frac{dR_{cyl}}{dt}\left(\frac{r^2}{2}-z^2\right).$$

Then the problem is transformed to the equivalent problem of bubble motion in an unbounded region, where the influence of the capillary wall is replaced by equivalent forces acting near the bubble wall, that is, the boundary conditions are taken into account. For this purpose, the velocity potential is represented as follows:

$$\varphi=\varphi_{bub}+\varphi_{no\,bub}.$$

In this case, when considering the bubble development in the initial stage (when the bubble is still small compared to the size of the capillary) φ_{bub} tends to zero at large distances from the bubble, and the boundary conditions for φ_{bub} on the bubble wall are represented as follows:

$$p_0-\rho\frac{\partial\varphi_{bub}}{\partial t}-\rho\frac{\left|\nabla\varphi_{bub}\right|^2}{2}=\rho\frac{\partial\varphi_{no\,bub}}{\partial t}+$$
$$+\rho\frac{\left|\nabla\varphi_{no\,bub}\right|^2}{2}+\rho\left(\nabla\varphi_{no\,bub},\nabla\varphi_{bub}\right)+$$
$$+p_{b0}\left(\frac{2R_0^{\,3}}{\int_{-1}^{1}R^2(\cos\theta,t)\,d(\cos\theta)}\right)^{\gamma}-2\sigma k$$

$$\frac{\partial R}{\partial t}(\cos\theta,t)=$$
$$=\left(1+\frac{\frac{\partial R}{\partial\tau}\cos\theta}{R(\cos\theta,t)}\right)\left(\frac{\partial\varphi_{bub}}{\partial r}\sin\theta+\frac{\partial\varphi_{bub}}{\partial z}\cos\theta\right)-$$
$$-\frac{\partial R}{\partial\tau}\frac{\frac{\partial\varphi_{bub}}{\partial z}}{R(\cos\theta,t)}+\left(1+\frac{\frac{\partial R}{\partial\tau}\cos\theta}{R(\cos\theta,t)}\right)\cdot$$
$$\cdot\left(\frac{\partial\phi_{no\,bub}}{\partial r}\sin\theta+\frac{\partial\varphi_{no\,bub}}{\partial z}\cos\theta\right)-\frac{\partial R}{\partial\tau}\frac{\frac{\partial\varphi_{no\,bub}}{\partial z}}{R(\cos\theta,t)}$$

Since the bubble does not grow to a large size during a single period of oscillation, the boundary conditions were averaged over a sufficiently large number of oscillation periods:

$$\left\langle p_0-\rho\frac{\partial\varphi_{bub}}{\partial t}-\rho\frac{\left|\nabla\varphi_{bub}\right|^2}{2}\right\rangle=\rho\left\langle\frac{\left|\nabla\varphi_{no\,bub}\right|^2}{2}\right\rangle+$$
$$+\rho\left\langle\left(\nabla\varphi_{no\,bub},\nabla\varphi_{bub}\right)\right\rangle+$$
$$+\left\langle p_{b0}\left(\frac{2R_0^{\,3}}{\int_{-1}^{1}R^2(\cos\theta,t)\,d(\cos\theta)}\right)^{\gamma}-2\sigma k\right\rangle$$

$$\left\langle\frac{\partial R}{\partial t}(\cos\theta,t)\right\rangle=$$
$$=\left\langle\left(1+\frac{\frac{\partial R}{\partial\tau}\cos\theta}{R(\cos\theta,t)}\right)\left(\frac{\partial\varphi_{bub}}{\partial r}\sin\theta+\frac{\partial\varphi_{bub}}{\partial z}\cos\theta\right)-\right.$$
$$\left.-\frac{\partial R}{\partial\tau}\frac{\frac{\partial\varphi_{bub}}{\partial z}}{R(\cos\theta,t)}\right\rangle+\frac{3}{R_{cyl}}\frac{dR_{cyl}}{dt}\frac{\partial R}{\partial\tau}\sin^2\theta\cos\theta$$

At the initial stage of bubble development, when the magnitude is small enough and the bubble is still close to spherical shape, the boundary conditions are transformed to the following form:

$$\left\langle p_0-\rho\frac{\partial\varphi_{bub}}{\partial t}-\rho\frac{\left|\nabla\varphi_{bub}\right|^2}{2}\right\rangle=$$
$$=\rho\left\langle\frac{\partial\varphi_{no\,bub}}{\partial t}+\frac{\left|\nabla\varphi_{no\,bub}\right|^2}{2}\right\rangle+$$
$$+\left\langle p_{b0}\left(\frac{2R_0^{\,3}}{\int_{-1}^{1}R^2(\cos\theta,t)\,d(\cos\theta)}\right)^{\gamma}-2\sigma k\right\rangle$$

$$\left\langle\frac{\partial R}{\partial t}(\cos\theta,t)\right\rangle=\left\langle\left(1+\frac{\frac{\partial R}{\partial\tau}\cos\theta}{R(\cos\theta,t)}\right)\cdot\right.$$
$$\left.\cdot\left(\frac{\partial\varphi_{bub}}{\partial r}\sin\theta+\frac{\partial\varphi_{bub}}{\partial z}\cos\theta\right)-\frac{\partial R}{\partial\tau}\frac{\frac{\partial\varphi_{bub}}{\partial z}}{R(\cos\theta,t)}\right\rangle$$

That is, an equivalent additional pressure force acts near the walls of the bubble

$$\rho\left\langle \frac{\partial \varphi_{no\ bub}}{\partial t} + \frac{\left|\nabla \varphi_{no\ bub}\right|^2}{2} \right\rangle = \rho\left\langle \frac{1}{R_{cyl}} \frac{d^2 R_{cyl}}{dt^2} \cdot \left(\frac{r^2}{2} - z^2\right) - \left(\frac{1}{R_{cyl}}\frac{dR_{cyl}}{dt}\right)^2 \left(\frac{r^2}{2} - z^2\right) + \right.$$
$$\left. + \left(\frac{1}{R_{cyl}}\frac{dR_{cyl}}{dt}\right)^2 \frac{(r)^2 + (2z)^2}{2} \right\rangle =$$
$$= \rho\left\langle \frac{1}{R_{cyl}}\frac{d^2 R_{cyl}}{dt^2}\left(\frac{r^2}{2} - z^2\right) + 3z^2\left(\frac{1}{R_{cyl}}\frac{dR_{cyl}}{dt}\right)^2 \right\rangle \approx$$
$$\approx 3z^2 \rho \left\langle \left(\frac{1}{R_{cyl}}\frac{dR_{cyl}}{dt}\right)^2 \right\rangle.$$

That is, at $z \neq 0$ an equivalent overpressure arises that tends to "pull" the cavitation bubble along the z-axis. Since the magnitude of the additional pressure is proportional to the square of z, the bubble "pulls out" in an avalanche-like manner. Thus, the bubble shape can be approximated by a cylindrical surface (Figure 1).

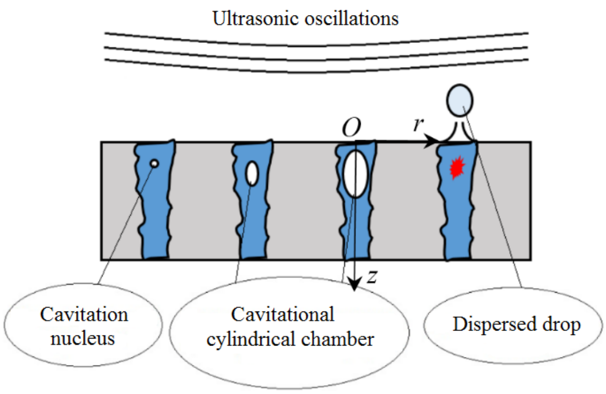


Figure 1. Approximation by a cylindrical surface

Using the equation of continuity and conservation of momentum, the mathematical statement of the problem of oscillations of the cylindrical cavity in the capillary volume is as follows (in the cylindrical coordinate system r is the distance from the symmetry axis of the cylindrical cavity, m; z is the coordinate along the symmetry axis of the cavity, m):

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial u}{\partial z} = 0,$$
$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r}\right) = -\frac{\partial p}{\partial r},$$
$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r}\right) = -\frac{\partial p}{\partial z},$$

where u, v - components of velocity of liquid surrounding the cavity along axis r and along axis z, respectively, m/s; p - pressure of liquid, Pa; ρ - density of liquid, kg/m^3.

It is assumed that the fluid flow is potential, and the problem is reduced to the Laplace equation for the velocity potential of the fluid:

$$u = \frac{\partial \varphi}{\partial r}; v = \frac{\partial \varphi}{\partial z};$$
$$\frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r} + \frac{\partial^2 \varphi}{\partial z^2} = 0.$$

The following form for solving the Laplace equation is proposed:

$$\varphi(r,z,t) = F_0(r,t) + F_1(r,t)G_1(z,t). \quad (1)$$

According to this, the boundary conditions on the wall of the cavitational cylindrical chamber and on the wall of the capillary are as follows:

$$\frac{\partial \varphi}{\partial r} = \frac{\partial F_0}{\partial r}(R(t),t) + \frac{\partial F_1}{\partial r}(R(t),t)G_1(z,t) = \frac{dR}{dt},$$
$$\frac{\partial F_1}{\partial r}(R(t),t) = 0,$$
$$\frac{\partial F_0}{\partial r}(R(t),t) = \frac{dR}{dt},$$
$$\frac{\partial F_1}{\partial r}(R_{cyl},t) = 0,$$
$$\frac{\partial F_0}{\partial r}(R_{cyl},t) = \frac{dR_{cyl}}{dt},$$

where R - radius of the cavitational cylindrical chamber, m; R_{cyl} - radius of capillary walls, m.
Substituting equation (1) into mass-momentum conservation equations with consideration of boundary conditions we obtain relations for functions F_0, F_1, G_1:

$$\frac{\partial^2 F_0}{\partial r^2} + \frac{1}{r}\frac{\partial F_0}{\partial r} + B = 0,$$
$$F_1 = 1,$$
$$\frac{\partial^2 F_0}{\partial r^2} + G_1\frac{\partial^2 F_1}{\partial r^2} + \frac{1}{r}\frac{\partial F_0}{\partial r} + G_1\frac{1}{r}\frac{\partial F_1}{\partial r} + F_1\frac{\partial^2 G_1}{\partial z^2} = 0,$$
$$G_1 = \frac{Bz^2}{2} + Dz + E,$$

where B is constant of liquid velocity gradient along coordinates, D is constant of longitudinal component of liquid velocity along axis z, E is constant of constant component of pressure.

Further, the dependence of radius of the cavitational cylindrical chamber on time is calculated. Boundary conditions for velocity on the cylinder walls:

$$u = \frac{\partial F_0}{\partial r}; \; u\left(R(t),t\right) = \frac{dR}{dt}; \; u\left(R_{cyl},t\right) = \frac{dR_{cyl}}{dt},$$

decision form:

$$u = -\frac{Br}{2} + \frac{C}{r},$$

where C is the liquid flow constant (volume of liquid per second per unit length of the capillary) is substituted into boundary conditions on the bubble wall and on the capillary wall:

$$-\frac{BR}{2} + \frac{C}{R} = \frac{dR}{dt}; \; -\frac{BR}{2}\frac{R_{cyl}}{R} + \frac{R}{R_{cyl}}\frac{C}{R} = \frac{dR_{cyl}}{dt},$$

solving the system of linear equations we find:

$$B = 2\frac{R\frac{dR}{dt} - R_{cyl}\frac{dR_{cyl}}{dt}}{R_{cyl}^2 - R^2};$$

$$C = RR_{cyl}\frac{R_{cyl}\frac{dR}{dt} - R\frac{dR_{cyl}}{dt}}{R_{cyl}^2 - R^2}.$$

The general form of the velocity potential, which satisfies the mass conservation equation:

$$\varphi = -\frac{Br^2}{4} + C\ln r + \frac{Bz^2}{2} + Dz + E,$$

substituting into the equation of conservation of momentum:

$$p = p_0 - \rho\frac{\partial}{\partial t}\left(-\frac{Br^2}{4} + C\ln r + \frac{Bz^2}{2} + Dz + E\right) - \\ -\rho\frac{\left(-\frac{Br}{2} + \frac{C}{r}\right)^2 + \left(B^2z^2 + 2DBz + D^2\right)}{2},$$

$$B = \frac{1}{t + t_0},$$

$$\frac{dD}{dt} = -\frac{2}{t + t_0}D,$$

$$D = \frac{D_0}{\left(t + t_0\right)^2},$$

$$\frac{1}{t + t_0} = -\frac{\frac{d}{dt}\left(R_{cyl}^2 - R^2\right)}{R_{cyl}^2 - R^2},$$

$$t_0 = -\frac{\left(R_{cyl0} + R_{cylA}\sin\left(\omega t_{start}\right)\right)^2 - R_0^2}{2\omega\left(R_{cyl0} + R_{cylA}\sin\left(\omega t_{start}\right)\right)} \cdot \\ \cdot\frac{1}{R_{cylA}\cos\left(\omega t_{start}\right)} - t_{start},$$

$$-\frac{\left(R_{cyl0} + R_{cylA}\sin\left(\omega t_{start}\right)\right)^2 - R_0^2}{2\omega\left(R_{cyl0} + R_{cylA}\sin\left(\omega t_{start}\right)\right)R_{cylA}\cos\left(\omega t_{start}\right)} \cdot \\ \cdot\left(R_{cyl}^2 - R_0^2\right) = S_{rel}.$$

We express the radius of the cavitational cylindrical chamber:

$$R^2 = \left(R_{cyl0} + R_{cylA}\sin\left(\omega t\right)\right)^2 + \\ + \frac{\frac{\left(\left(R_{cyl0} + R_{cylA}\sin\left(\omega t_{start}\right)\right)^2 - R_0^2\right)^2}{2\omega\left(R_{cyl0} + R_{cylA}\sin\left(\omega t_{start}\right)\right)R_{cylA}\cos\left(\omega t_{start}\right)}}{t - \frac{\left(R_{cyl0} + R_{cylA}\sin\left(\omega t_{start}\right)\right)^2 - R_0^2}{2\omega\left(R_{cyl0} + R_{cylA}\sin\left(\omega t_{start}\right)\right)R_{cylA}\cos\left(\omega t_{start}\right)} - t_{start}}$$

where t_{start} - start time of ultrasonic influence, s; R_{cyl0} - initial radius of capillary, m; R_{cylA} - amplitude of capillary radius oscillations, m.

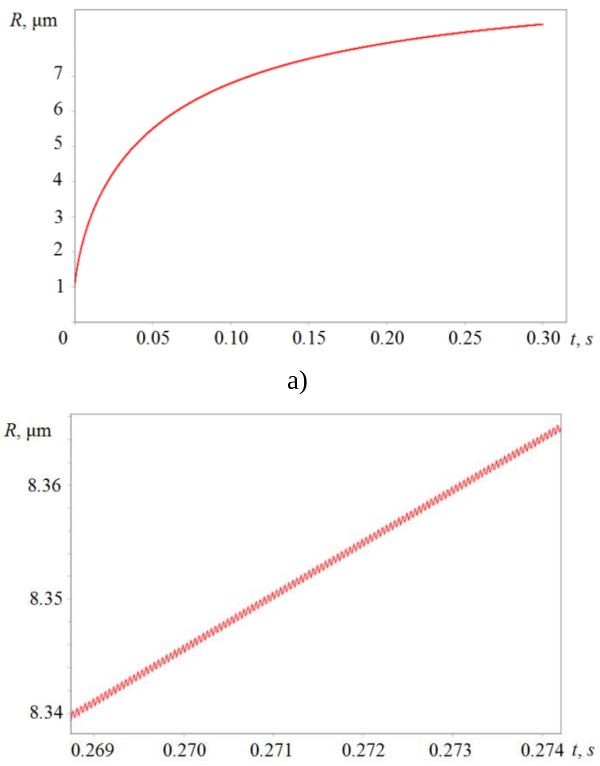
An example of dependence of the radius of a cavitational cylindrical chamber on time is shown in Figure 2 (in different scales).

From the equations obtained, it follows that at least 10 000 periods of oscillation are required to expand the bubble to a size sufficient for its collapse.

Taking into account the obtained results, a submodel for determining the dispersing capacity of the liquid from a unit surface area of the dried material is presented below.

3. SUB-MODEL OF SHOCK WAVE FORMATION AND DETERMINATION OF DISPERSING PERFORMANCE PER UNIT SURFACE AREA

A sub-model of shock wave formation and determination of dispersion performance per unit surface area assumes that the cavitational cylindrical chamber formed as a result of expansion turns out to be unstable. In this case the process has a random character (perturbations of velocity and pressure of liquid), which leads to chamber collapse due to the fact that rarefaction is created inside it (figure 2).



a)

b)

Figure 2. Dependence of the radius of a cavitational cylindrical chamber on time

Dispersing capacity ($m^3/(m^2*s)$ = m/s) as volume of dispersed liquid in 1 second from 1 m^2 of surface area is determined according to the following expression:

$$\Pi = \frac{\pi d^3}{6} \frac{E}{\sigma \pi d^2 \tau} wVn_{cap},$$

where E - shock wave energy at cavitational chamber collapse, J; d - drop diameter, m; τ - average time required for cavitational chamber collapse; n_{cap} - number of capillary cuts per unit surface area, m^{-2}; V - liquid volume flowing out for time τ from capillary under action of acoustic pressure, m^3; w - cavitational chamber formation probability per unit volume, m^{-3}.
Diameter of formed drop after slamming is determined by Rayleigh theory of jets collapse (Rayleigh-Plato instability). According to this theory, the most unstable perturbation wavelength is equal to $L = \frac{2\pi R}{0,697}$ (R - jet radius). According to this theory, the volume of a separated drop is equal to $V = \frac{2\pi^2 R^3}{0,697}$, and the drop diameter is equal to

$$d = 2\sqrt[3]{\frac{\frac{2\pi D^3}{0,697}}{8\frac{4}{3}}} = \frac{1}{2}\sqrt[3]{\frac{\frac{3\pi}{0,697}}{4}} D \approx 0,91D$$

(D - jet diameter, m).

The energy of the shock wave is determined according to the expression:

$$E = 4\pi R_{\min}{}^2 \tau_{sh} \frac{P_{\max}{}^2}{2\rho c}$$

where R - minimum radius of a chamber at collapse stage, m; - duration of cavitational chamber collapse, s; P_{max} - maximum pressure of shock wave at collapse stage, Pa.
Maximum pressure of shock wave and duration of chamber collapse are determined based on the Gilmore equation:

$$R\frac{\partial^2 R}{\partial t^2}\left(1 - \frac{\frac{\partial R}{\partial t}}{c}\right) + \frac{3}{2}\left(\frac{\partial R}{\partial t}\right)^2 \left(1 - \frac{\frac{\partial R}{\partial t}}{3c}\right) =$$
$$= H\left(1 + \frac{\frac{\partial R}{\partial t}}{c}\right) + \frac{\partial H}{\partial t}\frac{R}{c}\left(1 - \frac{\frac{\partial R}{\partial t}}{c}\right),$$

where R is instantaneous radius of cavitation cavity, m, H is enthalpy of liquid, m2/s2, c is local speed of sound in liquid phase, m/s.

In turn, probability of formation of a cavitational chamber in unit volume is proportional to concentration of cavitation nuclei in liquid.

Since capillary sizes are small (less than 1 mm), it is necessary to modify previously known model (Rosenberg, 1968; Golykh, 2021) to determine the concentration of cavitational chambers.

According to the proposed model, the probability of presence of n chambers in the volume V at the initial moment of time is determined according to the expression:

$$w_n = C_N^n \left(\frac{\Delta V}{V}\right)^n \left(1 - \frac{\Delta V}{V}\right)^{N-n} \approx$$
$$\approx \frac{N!}{n!(N-n)!}\left(\frac{n_{bub0}\Delta V}{N}\right)^n \left(1 - n_{bub0}\Delta V\right) \approx$$
$$\approx \frac{\left(n_{bub0}\Delta V\right)^n}{n!},$$

where n_{bub0} - is the concentration of chambers at the initial moment of time, m^{-3}; N is the number of bubbles in the volume V, *pcs*.

This calculation supposes that the surface of the material has a developed structure of honeycomb capillaries sized 20 µm and they are all filled with water.

According to the presented expression in small capillary volume there is most probably exactly one chamber at the initial moment of time. The probability of having two or more chambers at the same time in the same capillary is very small. At the same time, since the probability of having exactly one chamber at the initial moment of time is much less than unity, cavitation nuclei will not be present in all capillaries. The expression presented allows us to quantify the fraction of capillaries in the whole sample of dehydrated material in which cavitation germs will be present.

The found expression for the probability is used to calculate the dispersing capacity per unit surface area. Further we calculate dispersion productivity - the rate of moisture removal from a unit volume of material.

The sub-model of moisture removal due to dispersion in a cube-shaped material is based on the calculation of the acoustic field distribution in the sample volume according to the theory of linear acoustics:

$$\Delta P + k^2 P = 0,$$

$P = P_{out}(x, y, z)$ - on the surface of the sample S,

where P - complex amplitude of acoustic pressure inside the sample, *Pa*; P_{out} - complex amplitude of acoustic pressure in the air near the sample surface, *Pa*; k - wave number of the sample material, m^{-1} and calculation of integral dispersing capacity over the whole sample surface S divided by sample volume V (specific dispersing capacity is calculated):

$$\Pi_{total} = \frac{1}{V}\rho\int_S \Pi dS. \cdot$$

Further the dependences of dispersing productivity on modes of ultrasonic influence and sample size are presented.

4. RESULTS OF CALCULATIONS OF DISPERSING PRODUCTIVITY - MOISTURE REMOVAL

The obtained dependence of dispersion productivity on the sound pressure level for a sample of cubic shape (side equal to 15.6 mm) is shown in Figure 3.

There are three ranges of productivity of drying depending on sound pressure levels:

I (up to 150 dB) - weak growth of dispersing productivity with increasing sound pressure level;

II (from 150 to 170 dB) - sharp growth of dispersing productivity with increasing sound pressure level;

III (above 170 dB) - slow growth of dispersing productivity with increase of energy consumption for creation of ultrasonic oscillations.

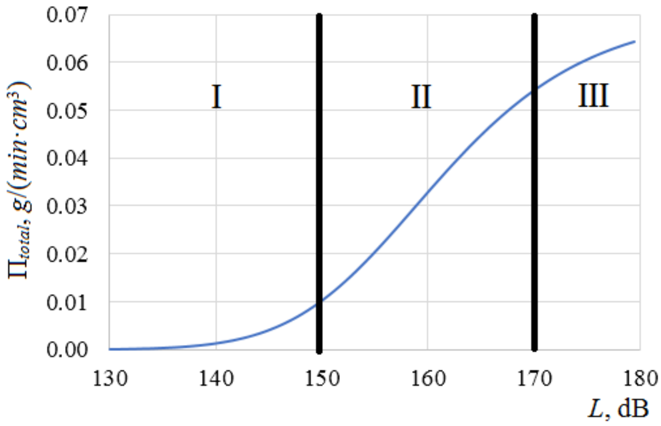


Figure 3. Dependence of dispersing productivity on level of sound pressure for sample with size 15.6 x 15.6 x 15.6 mm at frequency of ultrasonic oscillations 22 kHz

Drying in range I is not effective because liquid removal by dispersing is not significant and therefore the process is slightly different from convective drying.

In range II the removal of moisture without phase transition is more intense, reaching a maximum at 165-170 dB.

In range III the growth of dispersing productivity slows down, which is associated with a decrease in moisture diffusion through the capillaries to the surface of the material, as well as reaching the maximum size of the cavitation bubble, equal to the diameter of the capillary. At the same time energy costs to create such sound pressure levels increase significantly and their achievement is technically difficult. Therefore, the sound pressure level range of 150-170 dB is optimal for dewatering of capillary-porous bodies with resonant sample sizes.

Dispersing productivity for other values of samples sizes is presented in figure 4.

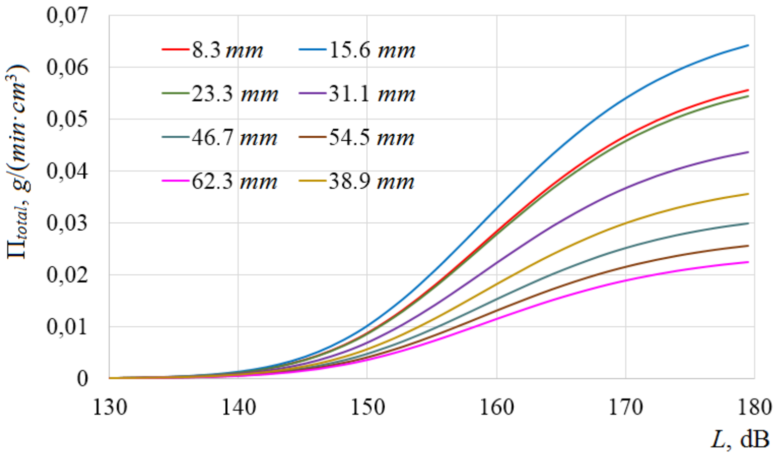


Figure 4. Dependences of dispersing productivity on sound pressure level for samples with size multiple of a half wavelength of ultrasonic oscillations

According to the presented dependencies for material samples with the size multiple of a half wavelength of ultrasonic oscillations the form of dependencies is the same for each of 3 ranges of sound pressure levels marked above. From a certain sound pressure level (145-150 dB for all sizes) begins a significant increase in productivity. This value should be considered the "threshold" at which the effectiveness of ultrasonic drying increases sharply and the use of ultrasonic exposure becomes effective. It is inexpedient to act with a sound pressure level higher than 170 dB, because the efficiency of the dehydration process decreases.

Thus, the course of the dependence of the dispersion rate on the sound pressure level is the same for samples of different sizes or thicknesses. But the absolute values of dispersion rate depend on the size (thickness) of the dried sample. Therefore, the dispersing productivity from the size of the material at different sound pressure levels was analyzed further (Figure 5).

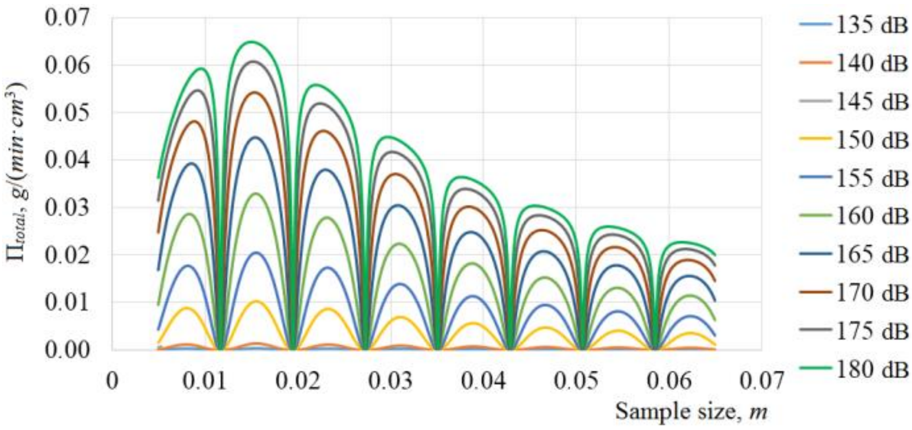


Figure 5. Dependencies of dispersing performance on the size of samples at different sound pressure levels

According to the dependencies presented, there are local optimums of dried material size at all sound pressure levels, at which the dispersing speed has a local maximum.

However, the greatest value of the dispersion rate reaches when the size or thickness of the material layer corresponds to the length of ultrasonic vibrations in air. At sizes smaller or larger than the wavelength in air, the process speed decreases.

Thus, it follows from the obtained results that the cavitation dispersion is the most effective from the material with the sample size equal or close to the wavelength λ of ultrasonic vibrations in air.

However, at material thickness 2λ the absorption of oscillations begins to affect, consequently, the pressure gradient increases, which leads to a sharp decrease in the efficiency of dispersion compared to the bulk material with a cube size 2λ. Further increase in the thickness of the extended form material will lead to leveling of the dispersing mechanism.

4. CONCLUSION

Although a conclusion may review the main points of the paper, do not replicate the abstract as the conclusion. A conclusion might elaborate on the importance of the work or suggest applications and extensions.

Theoretically shown the ability of ultrasonic oscillations to remove moisture without a phase transition due to its dispersion from the capillaries and pores of the material under the action of shock waves formed by cavitation bubbles of distorted cylindrical shape. The most probable mechanism of cavitation bubbles growth is proved.

A mathematical model describing the process of liquid removal without phase transition has been developed. It includes consideration of the following stages of liquid dispersion from capillaries and pores of material: the growth phase of the radius of a cavitational cylindrical chamber, the phase of shock wave formation, liquid dispersion.
The optimum range (150-170 dB) of ultrasonic influence levels was determined by numerical analysis of the model. The lower limit is determined by the appearance and development of cavitation dispergation of liquid, and the upper one - by energy efficiency of the dispergation process, above which the energy cost of creation of ultrasonic oscillations exceeds the effect of ultrasonic dispergation.

It was found that optimal conditions of exposure to ultrasonic oscillations on the dried material are realized when the size or thickness of the material layer corresponds to the length of ultrasonic oscillations in the air. If the size is smaller or larger, the efficiency of the drying process decreases.

The results obtained can be used in the development of ultrasonic dryers, providing the necessary modes and conditions of ultrasonic influence.

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