Vol. 04, No. 2 (2022) 101-114, doi: 10.24874/PES04.02.001

Proceedings on Engineering Sciences

www.pesjournal.net

LAPLACE ADOMIAN DECOMPOSITION AND NAVIER STOKES MODELLING FOR CLEAN AIR TECHNOLOGIES

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Received 11.09.2021.
Accepted 22.02.2022.
UDC – 303.725:614.712

Keywords:

Laplace Adomian decomposition; Navier Stokes equation; pollution; modeling; clean air

ABSTRACT

The study presents Laplace Adomian decomposition and Navier Stokes Modelling for clean air technologies analysis of some air pollution emission sources around the world. Previous literature showed that air pollution rates in the world emanated from different sources and affected the ecosystem and humans at large. Efforts made by researchers to curb the situation in the world at large yielded little result. This project developed a model and conducted simulations using pollution data obtained from the global air pollution database. From the results obtained, the Laplace Adomian decomposition and Navier Stokes method proved to be a fine method, especially with a large volume of data for predicting air pollution rates. The model can be employed in predicting pollution rate as a function of the driving parameters.

The validated results showed a decrease in optimal value of 0.3% which is a novelty. Finally, the total volume of pollution rates in the atmosphere (9.65x1010) when added with 11.57% by volume of hydroxyl radicals reduced the amount of pollution in the atmosphere as they tend to absorb these emissions to form compounds that are safe for human consumption and plants.

1. INTRODUCTION

Air pollution is seen as a major global threat causing deep impacts on human health and ecosystems (Franck et al., 2011; Peled, 2011). It is the major cause of premature death and disease, and the International Agency for Research on Cancer has classified it as carcinogenic. Particulate Matter (PM) has a huge impact on human health, thereby causing respiratory problems, bronchitis reduced lung functionality. Air pollution has a strong impact on ora and fauna, along with soil and water quality. The ecosystem in the world today, is affected because of its exposure to air pollutants (He et al., 2001). Nitrogen dioxide (NO2) affects both terrestrial and marine ecosystems by increasing the quantity of nutrient nitrogen in the natural systems, thereby causing eutrophication, and potentially leading to changes in species diversity or invasion of new species. Moreover,

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NO2 is partly responsible for the acidification of soils and waters. Ozone emissions affect crops, and forests by reducing plant growth.

Both ozone and particulate matter (PM) contributes to global warming (Hansen et al 2001). Human lives are also affected by pollution (OECD, 2016; Trippetta et al 2016). These emissions made international communities come up with policies that will reduce the emissions of pollutants. Air pollution nowadays is seen as the biggest threat to humans (Lorenzoni & Pidgeon, 2006) and major efforts have been done by policymakers to meet the World Health Organization air quality guidelines (Krzyzanowski & Cohen, 2008), as well as the United Nations Sustainable Development Goals (Griggs et al., 2013; Kumar et al., 2018; Yatkin & Bayram, 2007) Moreover, several studies have been performed on the relationship of pollution and its causes in many parts of the world (Alimissis et al., 2018; Lin, et al 2018; Elangasinghe et al., 2014) and these are aimed at reduction in pollution rates and its spread in the atmosphere. This project studied the different air pollution sources and developed a regression analysis model using several new parameters to aid in the prediction and mitigation of pollution rates. Air pollution is concerned with the introduction of chemicals, particulate matter, or biological materials into the atmosphere. These in turn cause discomfort to humans and other living organisms and the ecosystem (Bishoi et al., 2009). Because the introduction of chemicals and other polluting substances is not limited, models which relate the pollution rate, and any pollutants must be continuously renewed to reflect any potent indicators which can seriously increase pollution. Accordingly, this study examined existing pollution correlation models and incorporated novel pollution-causing indicators in a regression analysis using a pool of large data across several continents. The results contribute to the knowledge of its capacity in the prediction and mitigation of pollution rates. Due to the health implication of severely polluted environments, it is imperative to relate major drivers of pollution using a large volume of data. Such analysis will suggest to what extent these pollution indicators can be reduced to remedy the ecosystem and humans on its effect. In literature, very limited indicators are often related to the quantity of pollution rate and can severely thwart efforts have aimed at remediating pollution. In the light of the foregoing, the present study is justified under the empirical introduction of novel pollution-dependent indicators in a pool of large data to make a forecast-based regression model of pollution rate. It will help in striking a balance on the extent success can be recorded in reducing the pollution rate in the future.

2. LITERATURE REVIEW

Air pollution has many sources. Taking action to reduce the burden of disease from air pollution, therefore, begins with identifying the high-priority sources. More than 90% of people worldwide live in areas exceeding WHO guidelines for healthy air. More than half, live in areas that do not even meet WHOs least-stringent interim target for air quality. This research shall track air pollution exposures for three main pollutants: Fine particles (PM2.5), ozone, and household air pollution. Fine particle air pollution, referred to as ambient or outdoor PM2.5, comes from vehicle emissions, coal-burning power plants, industrial emissions, household energy use, and other sources. Long-term exposure to high levels of PM2.5 is harmful to one’s health (Serena et al., 2016).

2.1 Fine Particle Pollution

Air pollution has many sources. Taking action to reduce the burden of disease from air pollution, therefore, begins with identifying the high-priority sources. More than 90% of people worldwide live in areas exceeding WHO guidelines for healthy air. More than half, live in areas that do not even meet WHOs least-stringent interim target for air quality. This research shall track air pollution exposures for three main pollutants: Fine particles (PM2.5), ozone, and household air pollution. Fine particle air pollution, referred to as ambient or outdoor PM2.5, comes from vehicle emissions, coal-burning power plants, industrial emissions, household energy use, and other sources. Long-term exposure to high levels of PM2.5 is harmful to one’s health (Serena et al., 2016).

2.2 Vehicle Emission

The combustion of gasoline and diesel fuel in vehicle engines produces emissions of several potentially harmful substances. Also apparent is that evaporative emissions from refueling spills onto heated engine parts, and so on can equal emissions from the tailpipe. In addition, analyses have indicated that a significant source of emissions from vehicles is abrasion and wear of tires and metallic components, resulting in emissions of a variety of metals and carbon compounds. The gaseous and particulate pollutants to which motor vehicles contribute include carbon monoxide (CO), ozone (through its atmospheric precursor’s volatile organic compounds and nitrogen oxides [NOx]), fine particulate matter PM10 and PM2.5 (particles < 10 μm and < 2.5 μm in aerodynamic diameter, respectively), and nitrogen dioxide (Siegfried et al.,2011).

2.3 Coal-Burning Power Plants

Burning is the easiest and most economical option for the management of crop/biomass residues (Blas, 2014). Due to a lack of awareness or not availability of suitable technologies, it is generally practiced everywhere (Tripathi et al., 2013). It is also a significant source of aerosol in the atmosphere, having a potential impact on global air quality and the chemistry of climate. Bush burning is a detriment to the environment and health of mankind. The sources and effects of these pollutants were treated in this study.

2.4 Residential Heating

This is an essential energy service required by many people worldwide. Solid fuels heating consists of wood, forestry, agricultural residues, and even garbage. Currently, most burning of solid fuels for space heating is done in devices that incompletely combust the fuel owing to their low combustion temperature and other limitations. In both cases, most of the emissions end up in the atmosphere and contribute to outdoor air. Wood heating, while still a common practice even in some urban areas, has not received the same attention as coal, although it is also a major source of ambient air pollution during the heating season in nearly all parts of the world where wood is available. Some families revert to heating with solid fuels (such as discarded furniture, wood scrap, and coal) in response to economic hardship; this has happened recently in Greece and other European countries (Sinan et al., 2017).

2.5 Residential Heating

The industrial development of any country is the foundation stone of its prosperity provided it is not destructive. With the advancement of various technologies and industries, mankind will show great interest in natural sources by which they get an excessive amount of energy for their use. Now a day's industries play an important role for mankind and it is one of the major sources of atmospheric pollution which produces a huge amount of waste, which is released into the environment, and this is a major issue. Many industries like fertilizers, drugs, paper, cement, petroleum refineries, acid plants, plastic industries are responsible for about 22 percent of atmospheric pollution, such pollutants which are produced by this industry are various types of organic, inorganic compounds, gaseous oxides, (NOx, SO2) SPM and trace metal, etc. All such pollutants are hazardous to the human body.

2.6 The Ozone Layer

Particularly, ground-level ozone (O3) represents a major air pollution problem, both for public health and for the environment. Ozone is a reactive oxidant that forms in trace amounts in two parts of the atmosphere: the stratosphere (the layer 20-30 km above the earth's surface) and the troposphere (ground-level to 15 km). Stratospheric ozone, also known as "the ozone layer", is formed naturally and shields life on earth from the harmful effects of the sun's ultraviolet radiation.

Near the earth's surface, ground-level ozone can be harmful to human health and plant life and is created in part by pollution from man-made (anthropogenic) and natural (biogenic) sources. Tropospheric ozone is one of the most preponderant air pollutants in urban areas. An ozone level above some well-known threshold causes negative effects on biotic health. Indeed, tropospheric ozone is an irritating and reactive gas that is rather harmful to human health and affects other important parts of our daily life such as climate, farming, tourism, etc. Moreover, it is responsible for increases in mortality rates during episodes of high concentrations. In the light of the health effects of ground-level ozone, an accurate ozone alert forecasting system is necessary to warn the public before the ozone reaches a dangerous level. Ozone is not directly emitted by human activities. In the troposphere, it is a secondary pollutant whose formation depends on a complex cycle. Ozone is produced by atmospheric photochemical reactions that need solar radiation. Its production is led by volatile organic compounds (which include hydrocarbons) and nitrogen oxides concentrations, both emitted by anthropogenic activities.

3. DEVELOPMENT OF EMPIRICAL MODEL FOR DIFFERENT POLLUTION SOURCES

Biswanath et al., (2009) used the factor analysis method to predict the air quality index. Their analysis was based on daily emissions. They used multiple machine learning frameworks to forecast air pollution. They used a statistical approach to forecast ozone concentration. In other to achieve their goal they used a support vector regression model and random forest to predict the amount of ozone in the atmosphere and they used the Gaussian elimination method to predict PM2.5 and ozone 03.

3.1 Chemical Reactions of these pollutants: Ozone

The chemical formula for ozone is O3. It is formed in the earth's stratosphere in two reactive processes. The sun breaks the oxygen molecule into two oxygen atoms which then collide with another oxygen atom to form three atoms of oxygen. These reactions continue to occur whenever ultraviolet radiation is present in the earth's stratosphere. Besides its natural occurrence in the atmosphere, ozone could also be prepared in the laboratory. Near the earth's surface (Tropospheric zone) ozone is produced by a chemical reaction involving naturally occurring gases and gases from air pollution. Ozone production normally is from hydrocarbons and nitrogen oxides as well as ozone itself and all require the presence of sunlight to complete their reaction. Fossil fuel combustion is the primary source of pollutant gases that leads to the formation of ozone.

$O2 \quad + \text{UV sunlight} \rightarrow 2(O)$
$2x(O+O2+M) \rightarrow 03+M$
$\text{Net } 3O2 + \text{UV sunlight} \rightarrow 2(O3)$

Ozone in the atmosphere can be reduced by reacting with hydroxyl radicals to form water and oxygen which are both safe for humans.

H2O+0 UV Sunlight→2OH(Hydroxyl group
OH+O3→H2O+O2

3.2 Sulphur dioxide (SO2)

Sulfur dioxide would be removed from the atmosphere by reacting its emission with sodium hydroxide, this would release sodium sulfide which is not harmful to human health and water as shown in table 1-5.

Combining both would go a long way in cleaning the atmosphere and making the environment safe. The reaction is depicted below:

SO2(g) + 2NaOH(aq) → Na2CO3(aq)+ H2O (l)

3.3 Nitrogen dioxide (NO2)

The sunlight reacts with nitrogen dioxide to release water and nitric oxide which expands the blood vessels, reduces cholesterol, reduces plague growth, and increases blood flow.

NO2 + sunlight → NO + O

3.4 Carbon monoxide (CO)

Carbon monoxide reacts with hydroxyl radicals to yield carbon dioxide and hydrogen.

The carbon dioxide released into the atmosphere is safe for both humans and plants. The reaction of these compounds is represented below.

CO + OH →CO2 + H

Table 1. Emissions into the atmosphere

Pollutants	Natural origin	Anthropogenic origin
Carbon monoxide (CO)		$3.5\text{x}10^{8}$
Sulfur dioxide (SO2)	$1.4\text{x}10^{8}$	$1.45\text{x}10^{8}$
Nitrogen dioxide (NO2)	$1.4\text{x}10^{8}$	$(1.5\text{-}2.0)\ \text{x}10^{7}$
Particulate Matter	$(7.7\text{-}22.0)\ \text{x}10^{10}$	$(9.6\text{-}26.0)\ \text{x}10^{10}$
Ozone (O3)	$2.0\ \text{X}\ 10^{9}$	
Polyvinyl chloride materials		$2.0\ \text{X}\ 10^{6}$

Table 2. Major Components of Dry Clean Air

S/N	Gas	Molecular Mass	Relative amount in dry air % by volume.	Relative amount in dry air % by weight.	The relative amount of hydroxyl ion radicals by % Volume	The relative amount of hydroxyl ion radicals by % Volume
1	**Ozone(O3)**	48	0.00004	0.00007	0.0003	11.57
2	**Nitrogen(N2)**	28.02	78.08	75.53	0.0003	11.57
3	**Carbon**	44.01	0.033	0.05	0.0003	11.57
4	**dioxide (CO2)** **Nitrous**	44.02	0.00005	0.00008	0.0003	11.57
5	**oxide(N2O)** **Oxygen(O2)**	32	20.95	23.14	0.0003	11.57
6	**Hydrogen(H2)**	2.02	0.00005	0.000003	0.0003	11.57

3.5 Laplace Adomian Decomposition Method Air Pollution Contents

Air pollutants comprise carbon monoxide (CO), nitrogen oxide (NO and NO_2), Sulphur dioxide (SO_2), Ozone (O_3), particulate matter, and lead (Pb). The model is used to denote the Laplace adomian decomposition multi-dimensional Navier Stokes equation in an operator form.

$$D_t{}^{\beta}(f_{co}) + f_{co}\frac{\delta f_{co}}{\delta x_1} + f_{co}\frac{\delta f_{co}}{\delta x_2} + f_{co}\frac{\delta f_{co}}{\delta x_3} = \rho[\frac{\delta^2 f_{co}}{\delta_{x_1}{}^2} + \frac{\delta^2 f_{co}}{\delta_{x_2}{}^2} + \frac{\delta^2 f_{co}}{\delta_{x_3}{}^2}] \tag{1}$$

$$D_t{}^{\beta}(f_{NO}) + f_{NO}\frac{\delta f_{NO}}{\delta x_1} + f_{NO}\frac{\delta f_{NO}}{\delta x_2} + f_{NO}\frac{\delta f_{NO}}{\delta x_3} = \rho[\frac{\delta^2 f_{NO}}{\delta_{x_1}{}^2} + \frac{\delta^2 f_{NO}}{\delta_{x_2}{}^2} + \frac{\delta^2 f_{NO}}{\delta_{x_3}{}^2}] \tag{2}$$

$$D_t{}^{\beta}(f_{NO_2}) + f_{NO_2}\frac{\delta f_{NO_2}}{\delta x_1} + f_{NO_2}\frac{\delta f_{NO_2}}{\delta x_2} + f_{NO_2}\frac{\delta f_{NO_2}}{\delta x_3} = \rho[\frac{\delta^2 f_{NO_2}}{\delta_{x_1}{}^2} + \frac{\delta^2 f_{NO_2}}{\delta_{x_2}{}^2} + \frac{\delta^2 f_{NO_2}}{\delta_{x_3}{}^2}] \tag{3}$$

$$D_t{}^{\beta}\left(f_{SO_2}\right) + f_{SO_2}\frac{\delta f_{SO_2}}{\delta x_1} + f_{SO_2}\frac{\delta f_{SO_2}}{\delta x_2} + f_{SO_2}\frac{\delta f_{SO_2}}{\delta x_3} = \rho\left[\frac{\delta^2 f_{SO_2}}{\delta_{x_1}{}^2} + \frac{\delta^2 f_{SO_2}}{\delta_{x_2}{}^2} + \frac{\delta^2 f_{SO_2}}{\delta_{x_3}{}^2}\right] \qquad (4)$$

$$D_t{}^{\beta}\left(f_{O_3}\right) + f_{O_3}\frac{\delta f_{O_3}}{\delta x_1} + f_{O_3}\frac{\delta f_{O_3}}{\delta x_2} + f_{O_3}\frac{\delta f_{O_3}}{\delta x_3} = \rho\left[\frac{\delta^2 f_{O_3}}{\delta_{x_1}{}^2} + \frac{\delta^2 f_{O_3}}{\delta_{x_2}{}^2} + \frac{\delta^2 f_{O_3}}{\delta_{x_3}{}^2}\right] \qquad (5)$$

$$D_t{}^{\beta}\left(f_{pm}\right) + f_{pm}\frac{\delta f_{pm}}{\delta x_1} + f_{pm}\frac{\delta f_{pm}}{\delta x_2} + f_{pm}\frac{\delta f_{pm}}{\delta x_3} = \rho\left[\frac{\delta^2 f_{pm}}{\delta_{x_1}{}^2} + \frac{\delta^2 f_{pm}}{\delta_{x_2}{}^2} + \frac{\delta^2 f_{pm}}{\delta_{x_3}{}^2}\right] \qquad (6)$$

$$D_t{}^{\beta}\left(f_{pb}\right) + f_{pb}\frac{\delta f_{pb}}{\delta x_1} + f_{pb}\frac{\delta f_{pb}}{\delta x_2} + f_{pb}\frac{\delta f_{pb}}{\delta x_3} = \rho\left[\frac{\delta^2 f_{pb}}{\delta_{x_1}{}^2} + \frac{\delta^2 f_{pb}}{\delta_{x_2}{}^2} + \frac{\delta^2 f_{pb}}{\delta_{x_3}{}^2}\right] - \frac{1}{\rho}\frac{\delta\rho}{\delta x_3} \qquad (7)$$

With initial conditions:

$$f_{co}(x_1, x_2, x_3, 0) = f(x_1, x_2, x_3) \qquad (8)$$
$$f_{NO}(x_1, x_2, x_3, 0) = h(x_1, x_2, x_3) \qquad (9)$$
$$f_{NO_2}(x_1, x_2, x_3, 0) = g(x_1, x_2, x_3) \qquad (10)$$
$$f_{SO_2}(x_1, x_2, x_3, 0) = i(x_1, x_2, x_3) \qquad (11)$$
$$f_{O_3}(x_1, x_2, x_3, 0) = j(x_1, x_2, x_3) \qquad (12)$$
$$f_{pm}(x_1, x_2, x_3, 0) = k(x_1, x_2, x_3) \qquad (13)$$
$$f_{pb}(x_1, x_2, x_3, 0) = l(x_1, x_2, x_3) \qquad (14)$$

Applying the Laplace transform to equation (1-7)

$$\mathcal{L}[D_t{}^{\beta}(f_{co})] + \mathcal{L}[f_{co}\frac{\delta f_{co}}{\delta x_1} + f_{co}\frac{\delta f_{co}}{\delta x_2} + f_{co}\frac{\delta f_{co}}{\delta x_3}] = \mathcal{L}\rho\left[\frac{\delta^2 f_{co}}{\delta_{x_1}{}^2} + \frac{\delta^2 f_{co}}{\delta_{x_2}{}^2} + \frac{\delta^2 f_{co}}{\delta_{x_3}{}^2}\right] - \mathcal{L}[\frac{1}{\rho}\frac{\delta\rho}{\delta x_{c0}}] \qquad (15)$$

$$\mathcal{L}[D_t{}^{\beta}(f_{NO})] + \mathcal{L}[f_{NO}\frac{\delta f_{NO}}{\delta x_1} + f_{NO}\frac{\delta f_{NO}}{\delta x_2} + f_{NO}\frac{\delta f_{NO}}{\delta x_3}] = \mathcal{L}\rho[\frac{\delta^2 f_{NO}}{\delta_{x_1}{}^2} + \frac{\delta^2 f_{NO}}{\delta_{x_2}{}^2} + \frac{\delta^2 f_{NO}}{\delta_{x_3}{}^2}] - \mathcal{L}[\frac{1}{\rho} \qquad (16)$$

$$\mathcal{L}[D_t{}^{\beta}\left(f_{NO_2}\right)] + \mathcal{L}[f_{NO_2}\frac{\delta f_{NO_2}}{\delta x_1} + f_{NO_2}\frac{\delta f_{NO_2}}{\delta x_2} + f_{NO_2}\frac{\delta f_{NO_2}}{\delta x_3}] = \mathcal{L}\rho[\frac{\delta^2 f_{NO_2}}{\delta_{x_1}{}^2} + \frac{\delta^2 f_{NO_2}}{\delta_{x_2}{}^2} + \frac{\delta^2 f_{NO_2}}{\delta_{x_3}{}^2}] - \mathcal{L}[\frac{1}{\rho}\frac{\delta\rho}{\delta x_{NO_2}}] \qquad (17)$$

$$\mathcal{L}[D_t{}^{\beta}\left(f_{SO_2}\right)] + \mathcal{L}[f_{SO_2}\frac{\delta f_{SO_2}}{\delta x_1} + f_{SO_2}\frac{\delta f_{SO_2}}{\delta x_2} + f_{SO_2}\frac{\delta f_{SO_2}}{\delta x_3}] = \mathcal{L}\rho[\frac{\delta^2 f_{SO_2}}{\delta_{x_1}{}^2} + \frac{\delta^2 f_{SO_2}}{\delta_{x_2}{}^2} + \frac{\delta^2 f_{SO_2}}{\delta_{x_3}{}^2}] - \mathcal{L}[\frac{1}{\rho}\frac{\delta\rho}{\delta x_{SO_2}}] \qquad (18)$$

$$\mathcal{L}[D_t{}^{\beta}\left(f_{O_3}\right)] + \mathcal{L}[f_{O_3}\frac{\delta f_{O_3}}{\delta x_1} + f_{O_3}\frac{\delta f_{O_3}}{\delta x_2} + f_{O_3}\frac{\delta f_{O_3}}{\delta x_3}] = \mathcal{L}\rho[\frac{\delta^2 f_{O_3}}{\delta_{x_1}{}^2} + \frac{\delta^2 f_{O_3}}{\delta_{x_2}{}^2} + \frac{\delta^2 f_{O_3}}{\delta_{x_3}{}^2}] - \mathcal{L}[\frac{1}{\rho}\frac{\delta\rho}{\delta x_{O_3}}] \qquad (19)$$

$$\mathcal{L}[D_t{}^{\beta}\left(f_{pm}\right)] + \mathcal{L}[f_{pm}\frac{\delta f_{pm}}{\delta x_1} + f_{pm}\frac{\delta f_{pm}}{\delta x_2} + f_{pm}\frac{\delta f_{pm}}{\delta x_3}] = \mathcal{L}\rho[\frac{\delta^2 f_{pm}}{\delta_{x_1}{}^2} + \frac{\delta^2 f_{pm}}{\delta_{x_2}{}^2} + \frac{\delta^2 f_{pm}}{\delta_{x_3}{}^2}] - \mathcal{L}[\frac{1}{\rho}\frac{\delta\rho}{\delta x_{pm}}] \qquad (20)$$

$$\mathcal{L}[D_t{}^{\beta}\left(f_{pb}\right)] + \mathcal{L}[f_{pb}\frac{\delta f_{pb}}{\delta x_1} + f_{pb}\frac{\delta f_{pb}}{\delta x_2} + f_{pb}\frac{\delta f_{pb}}{\delta x_3}] = \mathcal{L}\rho\left[\frac{\delta^2 f_{pb}}{\delta_{x_1}{}^2} + \frac{\delta^2 f_{pb}}{\delta_{x_2}{}^2} + \frac{\delta^2 f_{pb}}{\delta_{x_3}{}^2}\right] - \mathcal{L}[\frac{1}{\rho}\frac{\delta\rho}{\delta x_{pb}}] \qquad (21)$$

Applying differentiation property of Laplace transform

$$\mathcal{L}(f_{co}) = \frac{f(x_1,x_2,x_3)}{s} - \frac{1}{s^{\beta}}\mathcal{L}\left[f_{co}\frac{\delta f_{co}}{\delta x_1} + f_{co}\frac{\delta f_{co}}{\delta x_2} + f_{co}\frac{\delta f_{co}}{\delta x_3}\right] + \frac{\rho}{s^{\beta}}\mathcal{L}\left[\frac{\delta^2 f_{co}}{\delta x_1{}^2} + \frac{\delta^2 f_{co}}{\delta x_2{}^2} + \frac{\delta^2 f_{co}}{\delta x_3{}^2}\right] - \frac{1}{s^{\beta}}\mathcal{L}\left[\frac{1}{\rho}\frac{\delta\rho}{\delta x_{co}}\right], \qquad (22)$$

$$\mathcal{L}(f_{NO}) = \frac{f(x_1,x_2,x_3)}{s} - \frac{1}{s^{\beta}}\mathcal{L}\left[f_{NO}\frac{\delta f_{NO}}{\delta x_1} + f_{NO}\frac{\delta f_{NO}}{\delta x_2} + f_{NO}\frac{\delta f_{NO}}{\delta x_3}\right] + \frac{\rho}{s^{\beta}}\mathcal{L}\left[\frac{\delta^2 f_{NO}}{\delta x_1{}^2} + \frac{\delta^2 f_{NO}}{\delta x_2{}^2} + \frac{\delta^2 f_{NO}}{\delta x_3{}^2}\right] - \frac{1}{s^{\beta}}\mathcal{L}\left[\frac{1}{\rho}\frac{\delta\rho}{\delta x_{NO}}\right], \qquad (23)$$

$$\mathcal{L}\left(f_{NO_2}\right) = \frac{f(x_1,x_2,x_3)}{s} - \frac{1}{s^{\beta}}\mathcal{L}\left[f_{NO_2}\frac{\delta f_{NO_2}}{\delta x_1} + f_{NO_2}\frac{\delta f_{NO_2}}{\delta x_2} + f_{NO_2}\frac{\delta f_{NO_2}}{\delta x_3}\right] + \frac{\rho}{s^{\beta}}\mathcal{L}\left[\frac{\delta^2 f_{NO_2}}{\delta x_1{}^2} + \frac{\delta^2 f_{NO_2}}{\delta x_2{}^2} + \frac{\delta^2 f_{NO_2}}{\delta x_3{}^2}\right] - \frac{1}{s^{\beta}}\mathcal{L}\left[\frac{1}{\rho}\frac{\delta\rho}{\delta x_{NO_2}}\right], \qquad (24)$$

$$\mathcal{L}\left(f_{O_3}\right) = \frac{f(x_1,x_2,x_3)}{s} - \frac{1}{s^{\beta}}\mathcal{L}\left[f_{O_3}\frac{\delta f_{O_3}}{\delta x_1} + f_{O_3}\frac{\delta f_{O_3}}{\delta x_2} + f_{O_3}\frac{\delta f_{O_3}}{\delta x_3}\right] + \frac{\rho}{s^{\beta}}\mathcal{L}\left[\frac{\delta^2 f_{O_3}}{\delta x_1{}^2} + \frac{\delta^2 f_{O_3}}{\delta x_2{}^2} + \frac{\delta^2 f_{O_3}}{\delta x_3{}^2}\right] - \frac{1}{s^{\beta}}\mathcal{L}\left[\frac{1}{\rho}\frac{\delta\rho}{\delta x_{O_3}}\right], \qquad (25)$$

$$\mathcal{L}\left(f_{SO_2}\right) = \frac{f(x_1,x_2,x_3)}{s} - \frac{1}{s^{\beta}}\mathcal{L}\left[f_{SO_2}\frac{\delta f_{SO_2}}{\delta x_1} + f_{SO_2}\frac{\delta f_{SO_2}}{\delta x_2} + f_{SO_2}\frac{\delta f_{SO_2}}{\delta x_3}\right] + \frac{\rho}{s^{\beta}}\mathcal{L}\left[\frac{\delta^2 f_{SO_2}}{\delta x_1{}^2} + \frac{\delta^2 f_{SO_2}}{\delta x_2{}^2} + \frac{\delta^2 f_{SO_2}}{\delta x_3{}^2}\right] - \frac{1}{s^{\beta}}\mathcal{L}\left[\frac{1}{\rho}\frac{\delta\rho}{\delta x_{SO_2}}\right], \qquad (26)$$

$$\mathcal{L}\left(f_{pm}\right) = \frac{f(x_1,x_2,x_3)}{s} - \frac{1}{s^{\beta}}\mathcal{L}\left[f_{pm}\frac{\delta f_{pm}}{\delta x_1} + f_{pm}\frac{\delta f_{pm}}{\delta x_2} + f_{pm}\frac{\delta f_{pm}}{\delta x_3}\right] + \frac{\rho}{s^{\beta}}\mathcal{L}\left[\frac{\delta^2 f_{pm}}{\delta x_1{}^2} + \frac{\delta^2 f_{pm}}{\delta x_2{}^2} + \frac{\delta^2 f_{pm}}{\delta x_3{}^2}\right] - \frac{1}{s^{\beta}}\mathcal{L}\left[\frac{1}{\rho}\frac{\delta\rho}{\delta x_{pm}}\right], \qquad (27)$$

$$\mathcal{L}\left(f_{pb}\right) = \frac{f(x_1,x_2,x_3)}{s} - \frac{1}{s^{\beta}}\mathcal{L}\left[f_{pb}\frac{\delta f_{pb}}{\delta x_1} + f_{pb}\frac{\delta f_{pb}}{\delta x_2} + f_{pb}\frac{\delta f_{pb}}{\delta x_3}\right] + \frac{\rho}{s^{\beta}}\mathcal{L}\left[\frac{\delta^2 f_{pb}}{\delta x_1{}^2} + \frac{\delta^2 f_{pb}}{\delta x_2{}^2} + \frac{\delta^2 f_{pb}}{\delta x_3{}^2}\right] - \frac{1}{s^{\beta}}\mathcal{L}\left[\frac{1}{\rho}\frac{\delta\rho}{\delta x_{pb}}\right], \qquad (28)$$

The Adomian solutions can be written as:

$$f_{co}(x_1, x_2, x_3, t) = \sum_{j=0}^{\infty} t_j \qquad (29)$$
$$f_{No}(x_1, x_2, x_3, t) = \sum_{j=0}^{\infty} u_j \qquad (30)$$
$$f_{NO_2}(x_1, x_2, x_3, t) = \sum_{j=0}^{\infty} v_j \qquad (31)$$
$$f_{SO_2}(x_1, x_2, x_3, t) = \sum_{j=0}^{\infty} w_j$$
$$f_{O_3}(x_1, x_2, x_3, t) = \sum_{j=0}^{\infty} x_j \qquad (32)$$
$$f_{pm}(x_1, x_2, x_3, t) = \sum_{j=0}^{\infty} y_j \qquad (33)$$
$$f_{pb}(x_1, x_2, x_3, t) = \sum_{j=0}^{\infty} z_j \qquad (34)$$

The nonlinear terms are defined by infinite series of Adomian polynomials,

$$N_{co}(f_{co}) = \sum_{j=0}^{\infty} A_j \qquad (35)$$
$$N_{No}(f_{NO}) = \sum_{j=0}^{\infty} B_j \qquad (36)$$
$$N_{NO_2}\left(f_{NO_2}\right) = \sum_{j=0}^{\infty} C_j \qquad (37)$$
$$N_{SO_2}\left(f_{SO_2}\right) = \sum_{j=0}^{\infty} D_j \qquad (38)$$
$$N_{O_3}\left(f_{O_3}\right) = \sum_{j=0}^{\infty} E_j \qquad (39)$$
$$N_{pm}\left(f_{pm}\right) = \sum_{j=0}^{\infty} F_j \qquad (40)$$

$$N_{pb}\big(f_{pb}\big) = \textstyle\sum_{j=0}^{\infty} G_j$$

$$A_j = \frac{1}{j!}[\frac{d^j}{d\lambda^j}[N_{co}\textstyle\sum_{i=0}^{\infty}(\lambda^j\, u_j)]]_{\lambda=0} \tag{41}$$

$$B_j = \frac{1}{j!}[\frac{d^j}{d\lambda^j}[N_{NO}\textstyle\sum_{i=0}^{\infty}(\lambda^j\, u_j)]]_{\lambda=0} \tag{42}$$

$$C_j = \frac{1}{j!}[\frac{d^j}{d\lambda^j}[N_{NO_2}\textstyle\sum_{i=0}^{\infty}(\lambda^j\, u_j)]]_{\lambda=0} \tag{43}$$

$$D_j = \frac{1}{j!}[\frac{d^j}{d\lambda^j}[N_{SO_2}\textstyle\sum_{i=0}^{\infty}(\lambda^j\, u_j)]]_{\lambda=0} \tag{44}$$

$$E_j = \frac{1}{j!}[\frac{d^j}{d\lambda^j}[N_{O_3}\textstyle\sum_{i=0}^{\infty}(\lambda^j\, u_j)]]_{\lambda=0} \tag{45}$$

$$F_j = \frac{1}{j!}[\frac{d^j}{d\lambda^j}[N_{pm}\textstyle\sum_{i=0}^{\infty}(\lambda^j\, u_j)]]_{\lambda=0} \tag{46}$$

$$G_j = \frac{1}{j!}[\frac{d^j}{d\lambda^j}[N_{pb}\textstyle\sum_{i=0}^{\infty}(\lambda^j\, u_j)]]_{\lambda=0} \tag{47}$$

Using LADM solutions in equations (22-28)

$$\begin{gathered}\mathcal{L}(\textstyle\sum_{j=0}^{\infty} t_{j+1})=\frac{(fx_1,x_2,x_3)}{s} - \frac{1}{s^\beta}\mathcal{L}\left(\frac{1}{\rho}\frac{\delta\rho}{\delta x_1}\right) - \\ \frac{1}{s^\beta}\mathcal{L}[(\textstyle\sum_{j=0}^{\infty} f_{co})\frac{\delta(\sum_{j=0}^{\infty} f\,co)}{\delta x_1} + (\textstyle\sum_{j=0}^{\infty} f_{co})\frac{\delta(\sum_{j=0}^{\infty} f\,co}{\delta x_2} + \\ (\textstyle\sum_{j=0}^{\infty} f_{co})\frac{\delta(\sum_{j-0}^{\infty} f\,co)}{\delta x_3} + \frac{\rho}{s^\beta}\mathcal{L}[\frac{(\delta^2(\sum_{j=0}^{\infty} f\,co)}{\delta {x_1}^2} + \\ \frac{(\delta^2(\sum_{j=0}^{\infty} f\,co)}{\delta {x_2}^2} + \frac{(\delta^2(\sum_{j=0}^{\infty} f\,co)}{\delta {x_3}^2}\end{gathered} \tag{48}$$

$$\begin{gathered}\mathcal{L}(\textstyle\sum_{j=0}^{\infty} t_{j+1})=\frac{(fx_1,x_2,x_3)}{s} - \frac{1}{s^\beta}\mathcal{L}\left(\frac{1}{\rho}\frac{\delta\rho}{\delta x_1}\right) - \\ \frac{1}{s^\beta}\mathcal{L}[(\textstyle\sum_{j=0}^{\infty} f_{NO})\frac{\delta(\sum_{j=0}^{\infty} f\,NO)}{\delta x_1} + (\textstyle\sum_{j=0}^{\infty} f_{NO})\frac{\delta(\sum_{j=0}^{\infty} f\,NO}{\delta x_2} + \\ (\textstyle\sum_{j=0}^{\infty} f_{NO})\frac{\delta(\sum_{j-0}^{\infty} f\,NO)}{\delta x_3} + \frac{\rho}{s^\beta}\mathcal{L}[\frac{(\delta^2(\sum_{j=0}^{\infty} f\,NO)}{\delta {x_1}^2} + \\ \frac{(\delta^2(\sum_{j=0}^{\infty} f\,NO)}{\delta {x_2}^2} + \frac{(\delta^2(\sum_{j=0}^{\infty} f\,NO)}{\delta {x_3}^2}\end{gathered} \tag{49}$$

$$\begin{gathered}\mathcal{L}(\textstyle\sum_{j=0}^{\infty} t_{j+1})=\frac{(fx_1,x_2,x_3)}{s} - \frac{1}{s^\beta}\mathcal{L}\left(\frac{1}{\rho}\frac{\delta\rho}{\delta x_1}\right) - \\ \frac{1}{s^\beta}\mathcal{L}[(\textstyle\sum_{j=0}^{\infty} f_{NO_2})\frac{\delta(\sum_{j=0}^{\infty} f\,NO_2)}{\delta x_1} + \\ (\textstyle\sum_{j=0}^{\infty} f_{NO_2})\frac{\delta(\sum_{j=0}^{\infty} f\,NO_2}{\delta x_2} + (\textstyle\sum_{j=0}^{\infty} f_{NO_2})\frac{\delta(\sum_{j-0}^{\infty} f\,NO_2)}{\delta x_3} + \\ \frac{\rho}{s^\beta}\mathcal{L}[\frac{(\delta^2(\sum_{j=0}^{\infty} f\,NO_2)}{\delta {x_1}^2} + \frac{(\delta^2(\sum_{j=0}^{\infty} f\,NO_2)}{\delta {x_2}^2} + \frac{(\delta^2(\sum_{j=0}^{\infty} f\,NO_2)}{\delta {x_3}^2}\end{gathered} \tag{50}$$

$$\begin{gathered}\mathcal{L}(\textstyle\sum_{j=0}^{\infty} t_{j+1})=\frac{(fx_1,x_2,x_3)}{s} - \frac{1}{s^\beta}\mathcal{L}\left(\frac{1}{\rho}\frac{\delta\rho}{\delta x_1}\right) - \\ \frac{1}{s^\beta}\mathcal{L}[(\textstyle\sum_{j=0}^{\infty} f_{SO_2})\frac{\delta(\sum_{j=0}^{\infty} f\,SO_2)}{\delta x_1} + (\textstyle\sum_{j=0}^{\infty} f_{SO_2})\frac{\delta(\sum_{j=0}^{\infty} f\,SO_2}{\delta x_2} + \\ (\textstyle\sum_{j=0}^{\infty} f_{SO_2})\frac{\delta(\sum_{j-0}^{\infty} f\,SO_2)}{\delta x_3} + \frac{\rho}{s^\beta}\mathcal{L}[\frac{(\delta^2(\sum_{j=0}^{\infty} f\,SO_2)}{\delta {x_1}^2} + \\ \frac{(\delta^2(\sum_{j=0}^{\infty} f\,SO_2)}{\delta {x_2}^2} + \frac{(\delta^2(\sum_{j=0}^{\infty} f\,SO_2)}{\delta {x_3}^2}\end{gathered} \tag{51}$$

$$\begin{gathered}\mathcal{L}(\textstyle\sum_{j=0}^{\infty} t_{j+1})=\frac{(fx_1,x_2,x_3)}{s} - \frac{1}{s^\beta}\mathcal{L}\left(\frac{1}{\rho}\frac{\delta\rho}{\delta x_1}\right) - \\ \frac{1}{s^\beta}\mathcal{L}[(\textstyle\sum_{j=0}^{\infty} f_{O_3})\frac{\delta(\sum_{j=0}^{\infty} f\,O_3)}{\delta x_1} + (\textstyle\sum_{j=0}^{\infty})\frac{\delta(\sum_{j=0}^{\infty} f\,O_3}{\delta x_2} + \\ (\textstyle\sum_{j=0}^{\infty} f_{O_3})\frac{\delta(\sum_{j-0}^{\infty} f\,O_3)}{\delta x_3} + \frac{\rho}{s^\beta}\mathcal{L}[\frac{(\delta^2(\sum_{j=0}^{\infty} f\,O_3)}{\delta {x_1}^2} + \\ \frac{(\delta^2(\sum_{j=0}^{\infty} f\,O_3)}{\delta {x_2}^2} + \frac{(\delta^2(\sum_{j=0}^{\infty} f\,O_3)}{\delta {x_3}^2}\end{gathered} \tag{52}$$

$$\begin{gathered}\mathcal{L}(\textstyle\sum_{j=0}^{\infty} t_{j+1})=\frac{(fx_1,x_2,x_3)}{s} - \frac{1}{s^\beta}\mathcal{L}\left(\frac{1}{\rho}\frac{\delta\rho}{\delta x_1}\right) - \\ \frac{1}{s^\beta}\mathcal{L}[(\textstyle\sum_{j=0}^{\infty} f_{pm})\frac{\delta(\sum_{j=0}^{\infty} fpm)}{\delta x_1} + (\textstyle\sum_{j=0}^{\infty} f_{pm})\frac{\delta(\sum_{j=0}^{\infty} fpm}{\delta x_2} + \\ (\textstyle\sum_{j=0}^{\infty} f_{pm})\frac{\delta(\sum_{j-0}^{\infty} fpm)}{\delta x_3} + \frac{\rho}{s^\beta}\mathcal{L}[\frac{(\delta^2(\sum_{j=0}^{\infty} fpm)}{\delta {x_1}^2} + \\ \frac{(\delta^2(\sum_{j=0}^{\infty} fpm)}{\delta {x_2}^2} + \frac{(\delta^2(\sum_{j=0}^{\infty} fpm)}{\delta {x_3}^2}\end{gathered} \tag{53}$$

$$\begin{gathered}\mathcal{L}(\textstyle\sum_{j=0}^{\infty} t_{j+1})=\frac{(fx_1,x_2,x_3)}{s} - \frac{1}{s^\beta}\mathcal{L}\left(\frac{1}{\rho}\frac{\delta\rho}{\delta x_1}\right) - \\ \frac{1}{s^\beta}\mathcal{L}[(\textstyle\sum_{j=0}^{\infty} f_{pb})\frac{\delta(\sum_{j=0}^{\infty} f\,pb)}{\delta x_1} + (\textstyle\sum_{j=0}^{\infty} f_{pb})\frac{\delta(\sum_{j=0}^{\infty} f\,pb}{\delta x_2} + \\ (\textstyle\sum_{j=0}^{\infty} f_{pb})\frac{\delta(\sum_{j-0}^{\infty} f\,pb)}{\delta x_3} + \frac{\rho}{s^\beta}\mathcal{L}[\frac{(\delta^2(\sum_{j=0}^{\infty} f\,pb)}{\delta {x_1}^2} + \\ \frac{(\delta^2(\sum_{j=0}^{\infty} f\,pb)}{\delta {x_2}^2} + \frac{(\delta^2(\sum_{j=0}^{\infty} f\,pb)}{\delta {x_3}^2}\end{gathered} \tag{54}$$

Applying Laplace transform to confirm the linearity of pollutants:

$$\begin{gathered}\mathcal{L}(t_0) = \frac{f(x_1,x_2,x_3)}{s} + \frac{1}{s^\beta}\mathcal{L}\left(\frac{1}{\rho}\frac{\delta\rho}{\delta x_{co}}\right), \\ \mathcal{L}(u_0) = \frac{f(x_1,x_2,x_3)}{s} + \frac{1}{s^\beta}\mathcal{L}\left(\frac{1}{\rho}\frac{\delta\rho}{\delta x_{NO}}\right), \\ \mathcal{L}(v_0) = \frac{f(x_1,x_2,x_3)}{s} + \frac{1}{s^\beta}\mathcal{L}\left(\frac{1}{\rho}\frac{\delta\rho}{\delta x_{NO_2}}\right), \\ \mathcal{L}(w_0) = \frac{f(x_1,x_2,x_3)}{s} + \frac{1}{s^\beta}\mathcal{L}\left(\frac{1}{\rho}\frac{\delta\rho}{\delta x_{SO_2}}\right), \\ \mathcal{L}(x_0) = \frac{f(x_1,x_2,x_3)}{s} + \frac{1}{s^\beta}\mathcal{L}\left(\frac{1}{\rho}\frac{\delta\rho}{\delta x_{O_3}}\right), \\ \mathcal{L}(y_0) = \frac{f(x_1,x_2,x_3)}{s} + \frac{1}{s^\beta}\mathcal{L}\left(\frac{1}{\rho}\frac{\delta\rho}{\delta x_{pm}}\right), \\ \mathcal{L}(z_0) = \frac{f(x_1,x_2,x_3)}{s} + \frac{1}{s^\beta}\mathcal{L}\left(\frac{1}{\rho}\frac{\delta\rho}{\delta x_{pb}}\right),\end{gathered} \tag{55}$$

$$\begin{aligned}\mathcal{L}(\sum_{j=0}^{\infty} t_{j+1}) = -\frac{1}{s^\beta}\mathcal{L}[(\sum_{j=0}^{\infty} f_{coj})&\frac{\delta(\sum_{j=0}^{\infty} f_{coj}}{\delta x_1} \\ &+ \sum_{j=0}^{\infty} f_{coj})\frac{\delta(\sum_{j=0}^{\infty} f_{coj}}{\delta x_2} \\ &+ \sum_{j=0}^{\infty} f_{coj})\frac{\delta(\sum_{j=0}^{\infty} f_{coj}}{\delta x_3} \\ &+ \frac{\rho}{s^\beta}\mathcal{L}[\frac{\delta^2\ (\sum_{j=0}^{\infty} f_{coj}}{\delta {x^2}_1} + \frac{\delta^2\ (\sum_{j=0}^{\infty} f_{coj}}{\delta {x^2}_2} \\ &+ \frac{\delta^2\ (\sum_{j=0}^{\infty} f_{coj}}{\delta {x^2}_3}]\end{aligned}$$

$$\begin{aligned}\mathcal{L}(\sum_{j=0}^{\infty} u_{j+1}) = -\frac{1}{s^\beta}\mathcal{L}[(\sum_{j=0}^{\infty} f_{NOj})&\frac{\delta(\sum_{j=0}^{\infty} f_{NOj}}{\delta x_1} \\ &+ \sum_{j=0}^{\infty} f_{NOj})\frac{\delta(\sum_{j=0}^{\infty} f_{NOj}}{\delta x_2} \\ &+ \sum_{j=0}^{\infty} f_{NOj})\frac{\delta(\sum_{j=0}^{\infty} f_{NOj}}{\delta x_3} \\ &+ \frac{\rho}{s^\beta}\mathcal{L}[\frac{\delta^2\ (\sum_{j=0}^{\infty} f_{NOj}}{\delta {x^2}_1} \\ &+ \frac{\delta^2\ (\sum_{j=0}^{\infty} f_{NOj}}{\delta {x^2}_2} + \frac{\delta^2\ (\sum_{j=0}^{\infty} f_{NOj}}{\delta {x^2}_3}]\end{aligned}$$

$$\begin{aligned}\mathcal{L}(\sum_{j=0}^{\infty} v_{j+1}) = -\frac{1}{s^{\beta}}\mathcal{L}[(\sum_{j=0}^{\infty} f_{NO_2 j}) & \frac{\delta(\sum_{j=0}^{\infty} f_{NO_2 j}}{\delta x_1} \\ & + \sum_{j=0}^{\infty} f_{NO_2 j}) \frac{\delta(\sum_{j=0}^{\infty} f_{NO_2 j}}{\delta x_2} \\ & + \sum_{j=0}^{\infty} f_{NO_2 j}) \frac{\delta(\sum_{j=0}^{\infty} f_{NO_2 j}}{\delta x_3} \\ & + \frac{\rho}{s^{\beta}}\mathcal{L}[\frac{\delta^2\ (\sum_{j=0}^{\infty} f_{NO_2 j}}{\delta {x^2}_1} \\ & + \frac{\delta^2\ (\sum_{j=0}^{\infty} f_{NO_2 j}}{\delta {x^2}_2} + \frac{\delta^2\ (\sum_{j=0}^{\infty} f_{NO_2 j}}{\delta {x^2}_3}]\end{aligned}$$

$$\begin{aligned}\mathcal{L}(\sum_{j=0}^{\infty} w_{j+1}) = -\frac{1}{s^{\beta}}\mathcal{L}[(\sum_{j=0}^{\infty} f_{SO_2 j}) & \frac{\delta(\sum_{j=0}^{\infty} f_{SO_2 j}}{\delta x_1} \\ & + \sum_{j=0}^{\infty} f_{SO_2 j}) \frac{\delta(\sum_{j=0}^{\infty} f_{SO_2 j}}{\delta x_2} \\ & + \sum_{j=0}^{\infty} f_{SO_2 j}) \frac{\delta(\sum_{j=0}^{\infty} f_{SO_2 j}}{\delta x_3} \\ & + \frac{\rho}{s^{\beta}}\mathcal{L}[\frac{\delta^2\ (\sum_{j=0}^{\infty} f_{SO_2 j}}{\delta {x^2}_1} \\ & + \frac{\delta^2\ (\sum_{j=0}^{\infty} f_{SO_2 j}}{\delta {x^2}_2} + \frac{\delta^2\ (\sum_{j=0}^{\infty} f_{SO_2 j}}{\delta {x^2}_3}]\end{aligned}$$

$$\begin{aligned}\mathcal{L}(\sum_{j=0}^{\infty} x_{j+1}) = -\frac{1}{s^{\beta}}\mathcal{L}[(\sum_{j=0}^{\infty} f_{O_3 j}) & \frac{\delta(\sum_{j=0}^{\infty} f_{O_3 j}}{\delta x_1} \\ & + \sum_{j=0}^{\infty} f_{O_3 j}) \frac{\delta(\sum_{j=0}^{\infty} f_{O_3 j}}{\delta x_2} \\ & + \sum_{j=0}^{\infty} f_{O_3 j}) \frac{\delta(\sum_{j=0}^{\infty} f_{O_3 j}}{\delta x_3} \\ & + \frac{\rho}{s^{\beta}}\mathcal{L}[\frac{\delta^2\ (\sum_{j=0}^{\infty} f_{O_3 j}}{\delta {x^2}_1} \\ & + \frac{\delta^2\ (\sum_{j=0}^{\infty} f_{O_3 j}}{\delta {x^2}_2} + \frac{\delta^2\ (\sum_{j=0}^{\infty} f_{O_3 j}}{\delta {x^2}_3}]\end{aligned}$$

$$\begin{aligned}\mathcal{L}(\sum_{j=0}^{\infty} y_{j+1}) = -\frac{1}{s^{\beta}}\mathcal{L}[(\sum_{j=0}^{\infty} f_{pmj}) & \frac{\delta(\sum_{j=0}^{\infty} f_{pmj}}{\delta x_1} \\ & + \sum_{j=0}^{\infty} f_{pmj}) \frac{\delta(\sum_{j=0}^{\infty} f_{pmj}}{\delta x_2} \\ & + \sum_{j=0}^{\infty} f_{pmj}) \frac{\delta(\sum_{j=0}^{\infty} f_{pm}}{\delta x_3} \\ & + \frac{\rho}{s^{\beta}}\mathcal{L}[\frac{\delta^2\ (\sum_{j=0}^{\infty} f_{pmj}}{\delta {x^2}_1} \\ & + \frac{\delta^2\ (\sum_{j=0}^{\infty} f_{pmj}}{\delta {x^2}_2} + \frac{\delta^2\ (\sum_{j=0}^{\infty} f_{pmj}}{\delta {x^2}_3}]\end{aligned}$$

$$\begin{aligned}&\mathcal{L}(\textstyle\sum_{j=0}^{\infty} z_{j+1}) = -\frac{1}{s^{\beta}}\mathcal{L}[(\textstyle\sum_{j=0}^{\infty} f_{pbj}) \frac{\delta(\sum_{j=0}^{\infty} f_{pbj}}{\delta x_1} + \\ &\textstyle\sum_{j=0}^{\infty} f_{pbj}) \frac{\delta(\sum_{j=0}^{\infty} f_{pbj}}{\delta x_2} + \textstyle\sum_{j=0}^{\infty} f_{pbj}) \frac{\delta(\sum_{j=0}^{\infty} f_{pb}}{\delta x_3} + \\ &\frac{\rho}{s^{\beta}}\mathcal{L}[\frac{\delta^2\ (\sum_{j=0}^{\infty} f_{pbj}}{\delta {x^2}_1} + \frac{\delta^2\ (\sum_{j=0}^{\infty} f_{pbj}}{\delta {x^2}_2} + \frac{\delta^2\ (\sum_{j=0}^{\infty} f_{pbj}}{\delta {x^2}_3}] \qquad (56)\end{aligned}$$

4. RESULTS AND DISCUSSION

The burden of disease caused by PM2.5 as shown in table 3 is influenced by a variety of factors, including changes in air pollution levels. Changes in healthcare access or medical treatments can help lower the number of people who die from diseases linked to pollution. Changes in a country's socioeconomic development affect the illness load over time as well. Changes in population size and age structure have the greatest influence on these trends in general. Even though air pollution exposures are declining, if a population is growing faster than exposures are falling, the overall attributable burden of disease can rise. Similarly, an ageing population will almost certainly experience a larger disease burden, as older people are more susceptible to diseases associated with air pollution. More than half of the increased mortality linked to PM2.5 exposure over the last decade are thought to be due to population expansion and ageing of the world population. When nitrogen oxides and volatile organic molecules combine in the presence of sunlight, ozone is produced as shown in table 4. At ground level, tropospheric ozone is a dangerous pollutant primarily caused by human activity. Ozone in the stratosphere (6–30 miles above the Earth's surface) is a naturally occurring protective layer that shields us from ultraviolet light. Parts per billion (ppb) are used to monitor ozone levels (ppb).GBD scientists statistically blend data from the following sources to predict worldwide ozone levels near ground level:There are over 8,800 air quality sensors, as well as nine global chemical transport models that combine data on emissions, chemical reactions, and meteorological variables to estimate the movement and concentration of pollutants at a fine geographic scale (1 kilometre by 1 kilometer). Burning any sort of fuel (coal, charcoal, wood, agricultural residue, dung, and liquid fuels like kerosene) for any purpose (cooking, heating, or lighting) can result in fine particulate matter (PM2.5), carbon monoxide, and other pollutants being released into the home. Cooking with solid fuels is a significant source of household air pollution, but it is not the only one. As a result, these studies are likely to understate household air pollution in some areas, particularly in colder climates in figures 1-5. The fraction of the population living in houses where solid fuels are used for cooking is reported as exposure to household air pollution as shown in table 5. These proportions are eventually transformed into fine particulate matter (PM2.5) exposures in the air for the purposes of calculating health consequences.

Table 3. P.M2.5 regional level emissions in some continents of the world (2017)

S/N	**Location**	**P.M $_{2.5}$(µg/m3)**
1	South Asia	84
2	South-East Asia	73.6
3	Asia	58.0
4	Eastern Mediterranean Region	57.3
5	North Africa and Middle East	54.7
6	G20	52.7
7	Africa	47.7
8	Global	46.0
9	Sub-Saharan Africa	44.7
10	Western Pacific Region	44.1
11	Southeast Asia, East Asia, and Oceania	42.9
12	Central Sub-Saharan Africa	42.6
13	World Bank Low Income	42.3
14	East Asia & Pacific	39.9
15	Eastern Sub-Saharan Africa	34.0
16	Central Asia	25.9
17	Southern Sub-Saharan Africa	24.5
18	Latin America	21.5

(Source: Health Effects Institute. 2019. State of Global Air 2019. Boston MA.)

Table 4. Regional Level Ozone Exposures (2017)

S/N	**Location**	**Ozone(O3)**
1	South Asia	60.9
2	South-East Asia	57.1
3	Asia	60.6
4	Eastern Mediterranean Region	61.3
5	North Africa and Middle East	60.8
6	G20	59.9
7	Africa	51.1
8	Global	57.4
9	Sub-Saharan Africa	49.9
10	Western Pacific Region	63.8
11	Southeast Asia, East Asia, and Oceania	61.3
12	Central Sub-Saharan Africa	59.0
13	World Bank Low Income	55.9
14	East Asia & Pacific	60.8
15	Eastern Sub-Saharan Africa	45.8
16	Central Asia	56.8
17	Southern Sub-Saharan Africa	45.8
18	Latin America	48.0
19	Central Europe, Eastern Europe, and Central Asia	53.3
20	European Region	54.0
21	Latin America and Caribbean	48.0
22	High-income Asia Pacific	56.1
23	America	51.3
24	Western Europe	53.8
25	Australasia	37.7

(Source: Health Effects Institute. 2019. State of Global Air 2019. Boston MA.)

Table 5. Regional level household emissions (2017)

S/N	**Location**	**Household**
1	South Asia	0.99
2	South-East Asia	0.99
3	Asia	0.99
4	Eastern Mediterranean Region	0.99
5	North Africa and Middle East	0.99
6	G20	0.98
7	Africa	0.98
8	Global	0.98
9	Sub-Saharan Africa	0.97
10	Western Pacific Region	0.97

Table 5. Regional level household emissions (2017) (continued)

S/N	Location	Household
11	Southeast Asia, East Asia, and Oceania	0.97
12	Central Sub-Saharan Africa	0.97
13	World Bank Low Income	0.97
14	East Asia & Pacific	0.96
15	Eastern Sub-Saharan Africa	0.96
16	Central Asia	0.96
17	Southern Sub-Saharan Africa	0.96
18	Latin America	0.96
19	Central Europe, Eastern Europe, and Central Asia	0.95
20	European Region	0.95
21	Latin America and Caribbean	0.93
22	High-income Asia Pacific	0.92
23	America	0.91
24	Western Europe	0.90
25	Australasia	0.89

(Source: Health Effects Institute. 2019. State of Global Air 2019. Boston MA.)

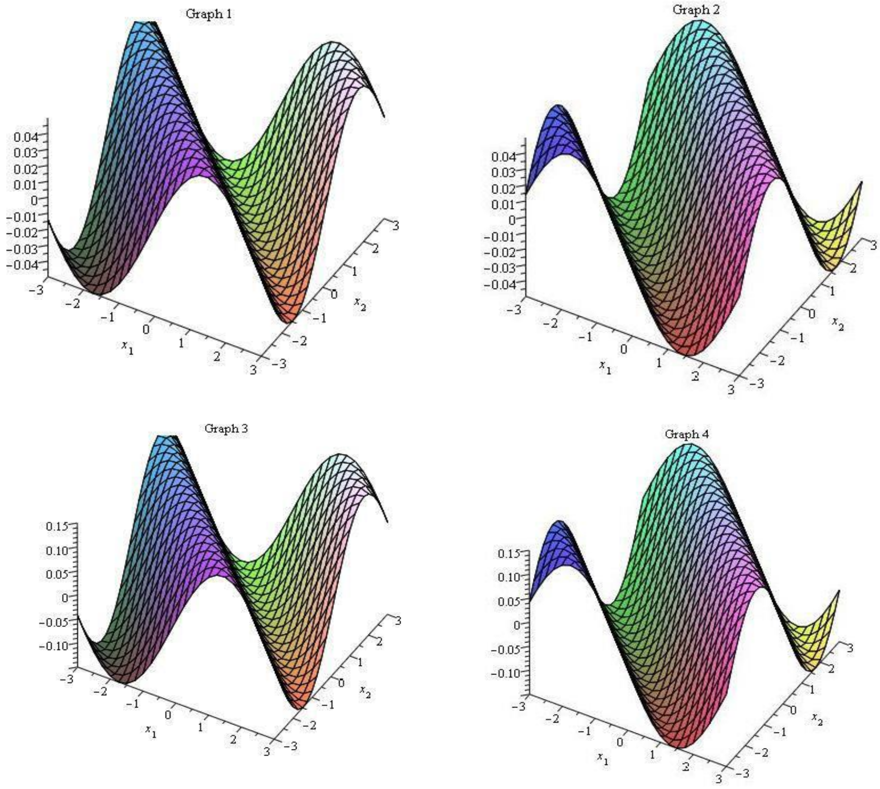

Figure 1. The velocity profiles f1, f2

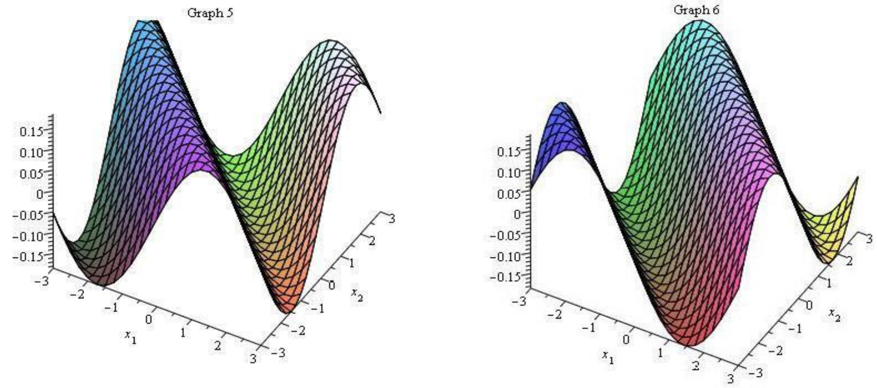

Figure 2. The velocity profiles f1, f2 of the NS equation

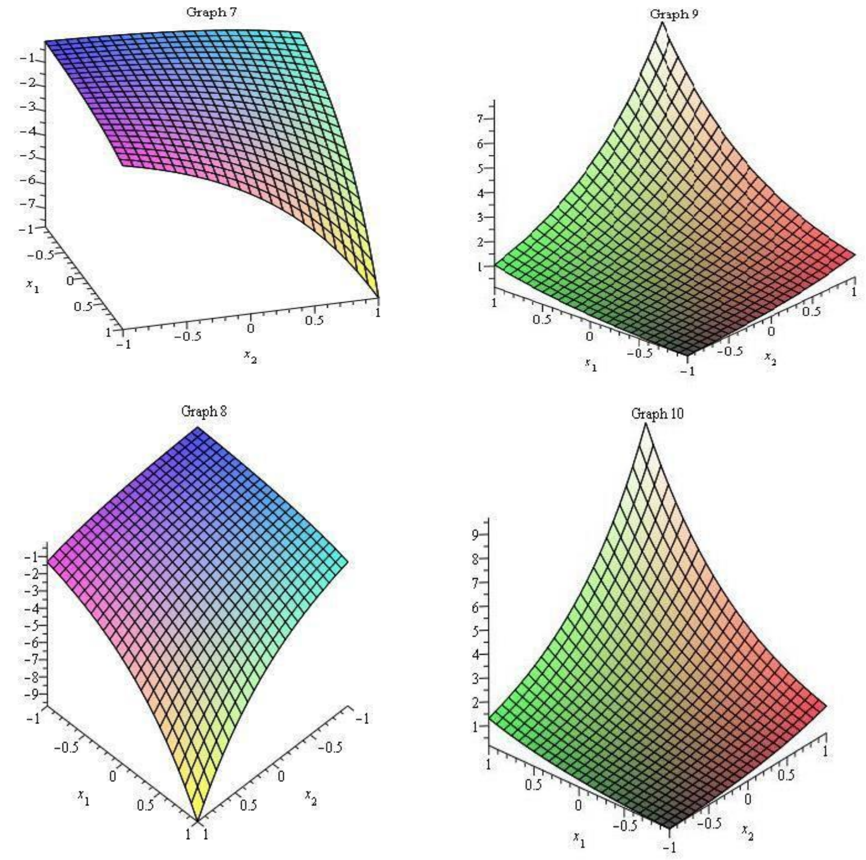

Figure 3. The velocity profiles f1, f2

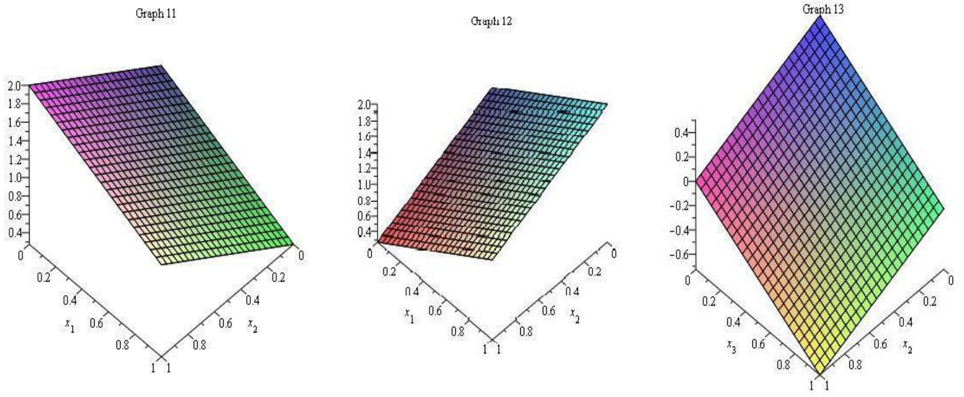

Figure 4. For example, 3, the velocity profiles f1, f2, f3

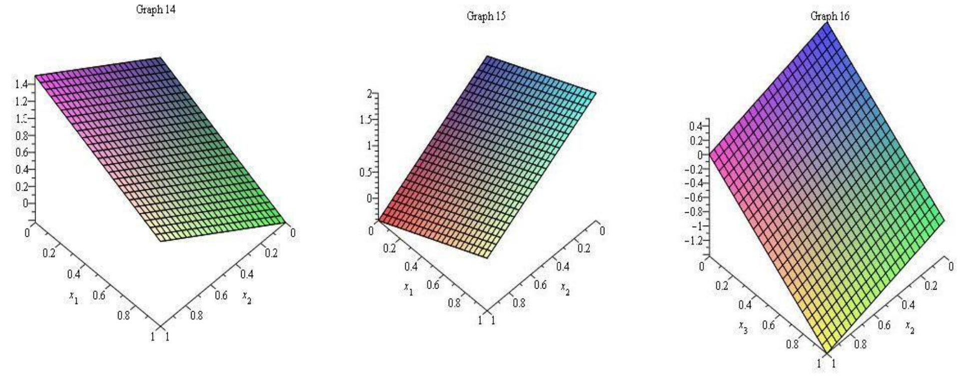

Figure 5. Annual average PM2.5 concentrations in 2017 relative to the WHO Air Quality Guideline

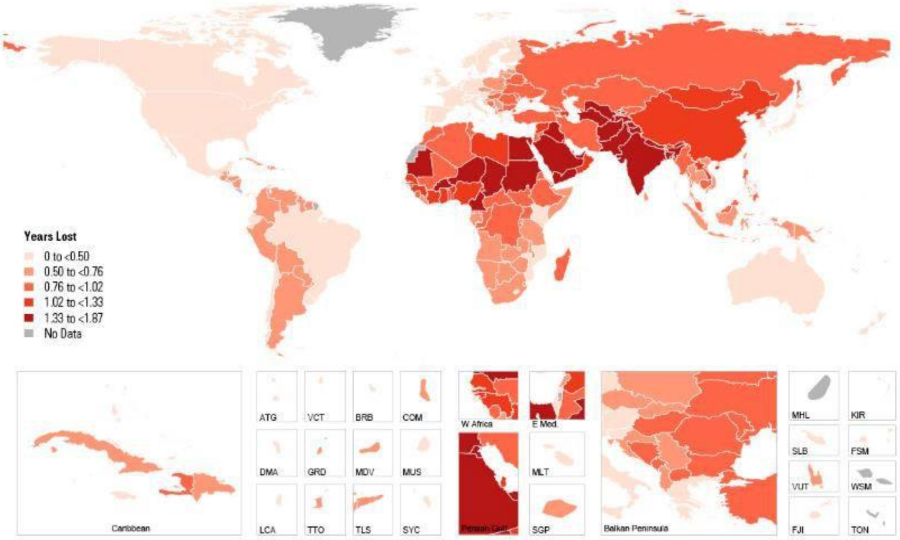

Figure 6. Global map of life expectancy loss attributable to existing levels of PM2.5 exposure in 2017

4.1 Simulated Results for Ozone

At ground level, where people live and breathe, Ozone (O3) could be seen as an irritant gas that seems to have harmful effects on human health. Ozone (O3), normally is formed from the reactions from sunlight in the presence of precursors chemicals such as nitrogen oxide and volatile organic compounds emitted from various industrial processes, power plants, motor vehicles, and other sources:

- Ozone concentrations were measured in units of parts per billion (ppb)
- Exposure to ozone for each country was measured as the population-weighed seasonal (8-hours maximum) concentration.
- The world's most developed regions have the highest ozone concentrations despite the extensive and successful air quality measures they've put in place in the past to ameliorate the emissions.
- The underdeveloped nations have seen their ozone rate increasing slowly but steadily because of warming temperatures and increased emissions of ozone precursors with industrialization.

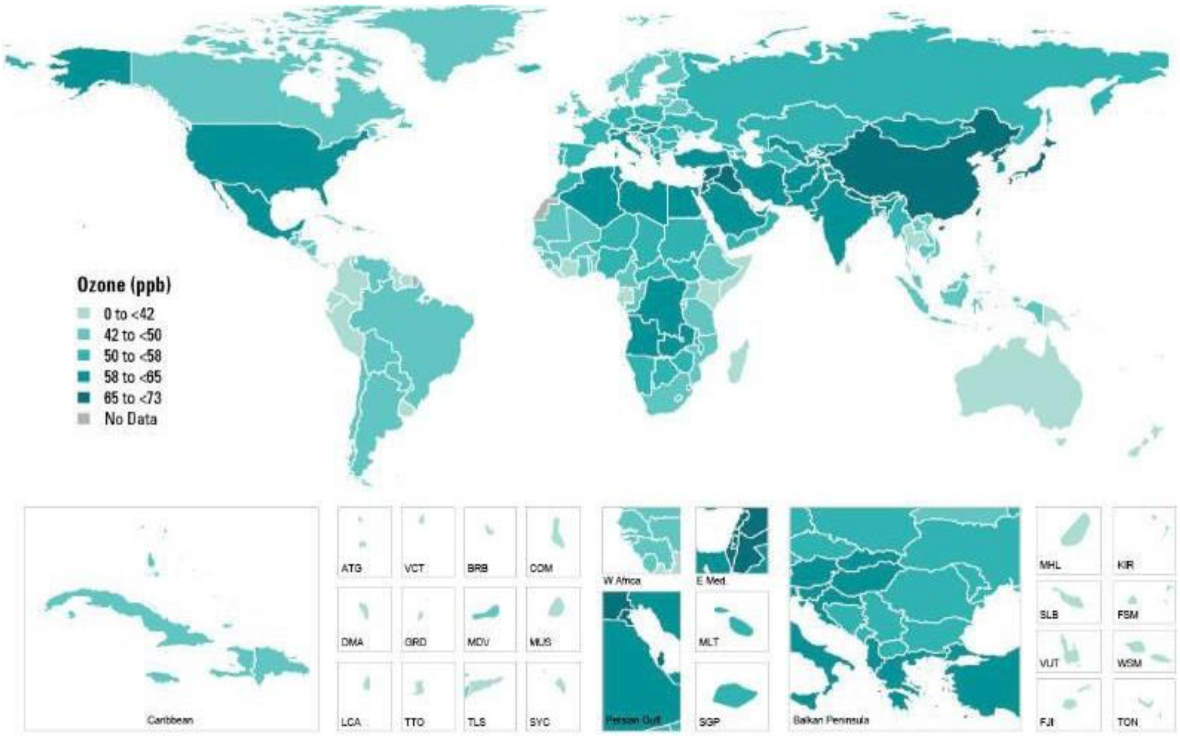

Figure 7. Population-weighted seasonal average (8-hour max) ozone concentrations in countries around the world in 2017

Figure 6 shows the percentage of exposure to household emissions in some continents in the world. In 2017, many people were exposed to household pollution emissions:

- Globally, nearly half of the world's population was affected. i.e-a total of 3.6billion people.
- India had 846 million people infected in 2017(about 60% of the population)
- In china,452million people were infected in 2017(32% of the population)
- In Bangladesh,124 million people were infected (79% of the population)
- In the democratic republic of congo,78 million people were infected (96% of the population)

5. CONCLUSION

The development of a new energy and air quality model has been a significant undertaking and represents an important step forward as a policy development tool. The inputs to the model system are numerous and the uncertainties are difficult to test in a comprehensive way. The results obtained showed that air pollution models is able to reproduce 2011 and 2012 concentrations of NOX, NO2, O3, PM10 and PM2.5 at spatial scales (see figures 1-7). Air pollution could be erased by letting citizens test their own air quality, growing an urban forest to clean the air, printing with inks made from polluted air, transforming smog into jewelry, cleaning the air with skyscrapers, introducing pollution vacuum cleaners, taking fossil fuels off the road for good, introducing trees in urban areas, setting the state for zero – emission vehicles, creating hydrogen fuel from air pollution, introducing air pollution sensors, smart streetlights and sensors, introducing anti-smog guns to shoot pollution down from the air and using google earth to track pollution in areas. Ongoing improvements in modeling of air pollution exposures, pollutants (beyond PM2.5) and disease outcomes, will further refine analyses. Specific examples include uncertainties in our understanding of PM10 non-exhaust emissions, which are assumed to increase pro rata with vehicle kilometers to 2050. This assumption may change as some private cars become lighter, are fitted with lower rolling resistant tires and use regenerative braking, whereas delivery vehicles become heavier, and as all vehicles are subject to increased city congestion and there are ongoing changes to the materials used in brakes and tire manufacture. Without regulation of these sources' future predictions should be considered with caution. Furthermore, the treatment of domestic wood burning emissions makes assumptions regarding the mix of wood burning appliances resulting in a 19% reduction in PM emissions per kilogram of wood burnt, because of the introduction of stoves complying with emission limits in the Ecodesign Directive (53% reduction in PM emissions compared with existing wood burners) and large pelletized domestic appliances (93% reduction in PM emissions compared with existing wood burners). Finally, although there will always be uncertainty in future predictions, the research aims were to provide alternative future scenarios, pointing out the potential for undesirable air pollution impacts within climate change policy and accepting that a large range of outcomes are possible. Laplace Adomian decomposition and Navier Stokes Modelling were used to predict pollution rates from different sources in the world, and this research justified its application.

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