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MODEL FOR MEASURING THE TECHNICAL EFFICIENCY OF THE PRODUCTION PROCESS IN THE AUTOMOTIVE INDUSTRY

Abstract: *The purpose of this paper is to point out the role and importance of measuring the efficiency of the production process in the automotive industry. The paper analyzes the production process in the automotive industry for a period of seven months. On the database obtained from the management information system, which was deliberately developed to control the process in the observed organization, the DEA output-oriented model was applied. The aim of the paper is to contribute to increasing the efficiency of the production process in the automotive industry based on the developed model, through analysis of key indicators of the quality control process.*

Keywords: *production process, automotive industry, measuring the technical efficiency, DEA model*

1. Introduction

The subject of the research in this paper is the production process of the covers for the protection of interior and exterior of cars during transport. The production process of one variant of covers in two shifts was analyzed in the period from January to July 2018. The data on the number of produced items per shifts, the number of returned products for finishing, the number of working days and workers that produced the items that were returned were taken from the management information system that is used in the observed organization.

For the measurement of technical efficiency of the production process an output-oriented DEA model was used.

The suggested model of measuring technical efficiency of the production process provides the support in optimization of decision making process for the organization management.

2. Methodology

Data envelopment analysis (DEA) is a mathematical, non-parameter approach for the calculation of efficiency that does not demand a specific functional form. It is used for the performance evaluation of decision making units (Decision Making Unit – DMU), in the way that several input variables come down to one ”virtual” input and several output variables come down to one “virtual” output, with the help of weighting coefficients.

The ratio DEA model, known as CCR model (Charnes et al., 1978) measures the efficiency of *j* DMU as maximum value of quotient weighted sum of outputs and weighted sum of inputs, that is:

$$(\max) h_k = \frac{\sum_{r=1}^{s} u_r y_{rk}}{\sum_{i=1}^{m} v_i x_{ik}} \qquad (1)$$

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s.t.

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i\, x_{ij}} \leq 1,\ \text{j=1,2,..,}n$$

$$u_r \geq 0,\ \text{r=1,2,..,}s$$
$$v_i \geq 0,\ \text{i=1,2,..,}m$$

Wereby:

h_k –k-th DMU relative efficiency;

n –DMU number to be compared;

m –number of input variables;

s – number of output variables;

u_r – output r weight coefficient;

v_i – input i weight coefficient.

CCR ratio model calculates the total technical (radial) efficiency into which pure technical efficiency and the efficiency as the consequence of different volumes of business are included. The value of objective function h_k is between 0 and 1. If the value h_k is equal to 1, k-th DMU is relatively efficient, and if it is less than 1, DMU_k is relatively non-efficient and the value h_k shows necessary percentage input decrease in order to become efficient.

The mentioned model of fractured linear programming has two operational forms, depending on orientation. The first form maximizes virtual output sum of j DMU, whereby its virtual input is equal to 1 and known as input-oriented model, while the second, used in this paper, minimizes the total virtual input, whereby virtual output is equal to 1, and is known as output-oriented model. Input-oriented results of efficiency are between 0 and 1, while the results oriented to output efficiency are in the range of 1 to infinity, whereby in both cases DMU_j whose efficiency are equal to 1, are relatively efficient. Important assumptions on which valid application of DEA model is based, are defined by the principle of *homogeneity* i.e. the similarity of decision making units, the property of *positivity* of input and output variables, the trait of *isotonicity* that implies that the increase of some input results in the same increase of outputs without the decrease of any other input, as well as the optimal number of input and output variables that completely measures the performance of decision making units and is mutual for all decision making units (in more details on practical application of DEA method see in: Dyson et al. 2001; Sarkis, 2002, 2007; Sherman & Zhu, 2006; Cooper et al, 2007; Cook et al, 2014; etc.).

3. Description and structuring of a problem

Starting from the research objective, for the measurement of production efficiency output-oriented CCR ratio model was chosen (CCR – O), whose corresponding dual form that is most often solved, the form, in general case is:

$$(\max)\ \emptyset_k + \varepsilon \left(\sum_{i=1}^{m} s_i^{\,-} + \sum_{r=1}^{s} s_r^{\,+}\right) \qquad (2)$$

s.t.

$$\sum_{j=1}^{m} x_{ij}\, \lambda_j + s_i^{\,-} = x_{ik} \qquad i = 1,2,\ldots,m$$
$$\sum_{j=1}^{n} y_{rj}\, \lambda_j - s_r^{\,+} = \emptyset y_{rk} \qquad r = 1,2,\ldots,s$$
$$\lambda_j,\ s_r^{\,+}, s_i^{\,-} \geq 0$$
$$j = 1,2,\ldots,n;\ i = 1,2,\ldots,m;\ r = 1,2,\ldots,s$$

Whereby $s_i^{\,-}$ and $s_r^{\,+}$ dual variables that speak on the necessary individual decrease of i-th input and increase of r-th output k-th DMU in order to become efficient. Dual variable λj represents dual weight that shows the importance that is assigned to DMUj (j = 1,2,..,n) when defining input-output mix of hypothetical composite unit with which DMUk will be directly compared. DMUk is technically efficient, if and only if, for optimal solution ($\lambda^*, s^{+*}, s^{-*}$, $\emptyset_k^{\,*}$) conditions have been met: $\emptyset_k^{\,*} = 1$; $s^{+*} =$

$0;\ s^{-*} = 0$. In output-oriented model relative efficiencies are equal or bigger than 1, whereby relatively efficient are those DMU_s whose efficiency is 1, and non-efficient are the ones whose efficiency is bigger than 1.

As suggested by numerous studies, the number DMU in the observed set should be double or triple bigger than the total number of inputs and outputs, since there is the danger that the most DMU will be classified as efficient precisely because of the characteristic of DEA to strive to display each unit as much better as possible (Charnes et al. 1994; Dyson, 2001; Sarkis, 2002; Cook et al. 2014; Subramanyam, 2017).

In the literature there can be found different rules on optimal number of input and output variables, as it is general rule $m+s <n/3$ *or* $m\times s <n$ and $m+s <n/2$ (Cooper, Seiford, & Tone, 2007). In that sense, in scientific and professional literature different approaches to the number of inputs and outputs are known and most often the correlation and regression analysis.

The choice of variables in this case was made on the basis of the interview with manager of the production process of the observed organization and was confirmed with the correlation analysis (table 3).

So that the assumptions on which formed DEA CCR-O model rests are:

- The observed time interval is January-July, 2018, per shifts A and B that at the same time make the set of observed DMUs;
- Input variables are: I1 – number of working days in month, I2 – number of returned products for finishing (table 1);
- Output variable is O1 – production (table 1).

Table 1. Values of input and output variables

DMU	I1	I2	O1
January A	18	1	7200
January B	18	4	8789
February A	18	9	3500
February B	18	1	500
March A	22	10	17950
March B	22	49	13800
April A	18	1	450
April B	18	18	7550
May A	21	14	10000
May B	21	7	10000
Jun A	21	13	13100
Jun B	21	13	10900
July A	22	27	34700
July B	22	19	26470

Table 2. Descriptive statistics

Variable	I1	I2	O1
max	22,0000	49,0000	34700,0000
min	18,0000	1,0000	450,0000
mean	20,0000	13,2857	11779,2143
SD	1,8397	12,7908	9462,9093

Table 3. Matrix of correlation coefficients

		(I)I1	**(I)I2**	**(O)O1**
(I)I1	**R**	**1,0000**		
	R Standard Error			
	t			
	p-value			
	H0 (5%)			
(I)I2	**R**	**0,6178**	**1,0000**	
	R Standard Error	0,0515		
	t	2,7218		
	p-value	0,0185		
	H0 (5%)	*rejected*		
(O)O1	**R**	**0,7682**	**0,5321**	**1,0000**
	R Standard Error	0,0342	0,0597	
	t	4,1568	2,1772	
	p-value	0,0013	0,0491	
	H0 (5%)	*rejected*	*rejected*	

4. Model results

Technical efficiency for each DMUs (shift) was estimated by dual model (2), by estimating the capacity of each shift in production maximization. Relative efficiency of the shift is estimated by the comparison of best practices observed during the analyzed period, in the range from January shift A to July shift B. The analysis of obtained results (table 4) shows that reference set DMU_s consists of shift A in January, March and July, whose technical efficiency is equal to 1, the utilization of working days was 100%, the number of returned products for finishing was practically minimum possible, all dual variables s_i^- and s_r^+ are equal to 0, so the aimed values of input and output variables are equal to realized (table 5). Other DMUs are technically non-efficient (least relative technical efficiency is realized in April, in shift A (0,0625). For the purpose of illustration, optimal solution for shift B in April is:

$$\emptyset^* = 0{,}3099;\ \lambda_{mart,A}{}^* = 0{,}241;\ \lambda_{jul,A}{}^* = 0{,}578;$$
$$s_1{}^- = 0;\ s_2{}^- = 0;\ s_1{}^+ = 24360;$$

Since $\lambda_{mart,A}{}^* > 0\ \ i\ \lambda_{jul,A}{}^* > 0$, reference set for $DMU_{april,B}$ is R_{2009} $\{mart, A; jul, A,\ \}$. Via these referential values λ^* it is possible to calculate the aimed value of output variable O1, for which shift B in April was technically efficient, while for input variables I1 and I2 the aimed values are identical to the realized ones. That is, for output variable O1, it follows:

$$O_{1_{april,B}}{}^* = \lambda_{mart,A}{}^* \times O_{1_{mart,A}} + \lambda_{jul,A}{}^* \times O_{1_{\mathrm{jul,A}}},$$
$$O_{1_{april,B}}{}^* = 0{,}241\times17950+0{,}578\times34700 = 24382{,}55$$

Which means that the shift B in April had to increase the production by 222,7% (from 7550 to approximately 24380 items) in order to be on the level of best reference practice, that is to be technically efficient. Similar calculation and analysis can be performed for other DMUs as well, i.e. shifts.).

Table 4. Relative technical efficiency per shift

DMU	Score	Rank		Reference (Lambda)		
January A	1	1	January A	1		
January B	0,8511	5	January B	0,582	Mart A	0,342
February A	0,2259	12	February A	0,77	Jul A	0,048
February B	0,0694	13	February B	1		
March A	1	1	March A	1		
March B	0,3977	10	March B	1		
April A	0,0625	14	April A	1		
April B	0,3099	11	April B	0,241	Jul A	0,578
May A	0,4646	9	May A	0,693	Jul A	0,262
MayB	0,6906	6	MayB	0,354	Mart A	0,665
Jun A	0,6378	7	Jun A	0,751	Jul A	0,203
Jun B	0,5307	8	Jun B	0,751	Jul A	0,203
July A	1	1	July A	1		
July B	0,987	4	July B	0,471	Jul A	0,529

Table 5. Realized and aimed values of input and output variables

			I1		I2			O1		
No.	DMU	Score	Data	Projection	Data	Projection	Diff.(%)	Data	Projection	Diff.(%)
1	January A	1	18	18	1	1	0	7200	7200	0
2	January B	0,851	18	18	4	4	0	8789	10327	17,50
3	February A	0,226	18	18	9	9	0	3500	15492	342,6
4	February B	0,069	18	18	1	1	0	500	7200	1340
5	March A	1	22	22	10	10	0	17950	17950	0
6	March B	0,398	22	22	49	27	-44,9	13800	34700	151,5
7	April A	0,063	18	18	1	1	0	450	7200	1500
8	April B	0,309	18	18	18	18	0	7550	24360	222,7
9	May A	0,465	21	21	14	14	0	10000	21523	115,2
10	MayB	0,691	21	21	7	7	0	10000	14480	44,80
11	Jun A	0,638	21	21	13	13	0	13100	20538	56,78
12	Jun B	0,531	21	21	13	13	0	10900	20538	88,42
13	July A	1	22	22	27	27	0	34700	34700	0
14	July B	0,987	22	22	19	19	0	26470	26818	1,313

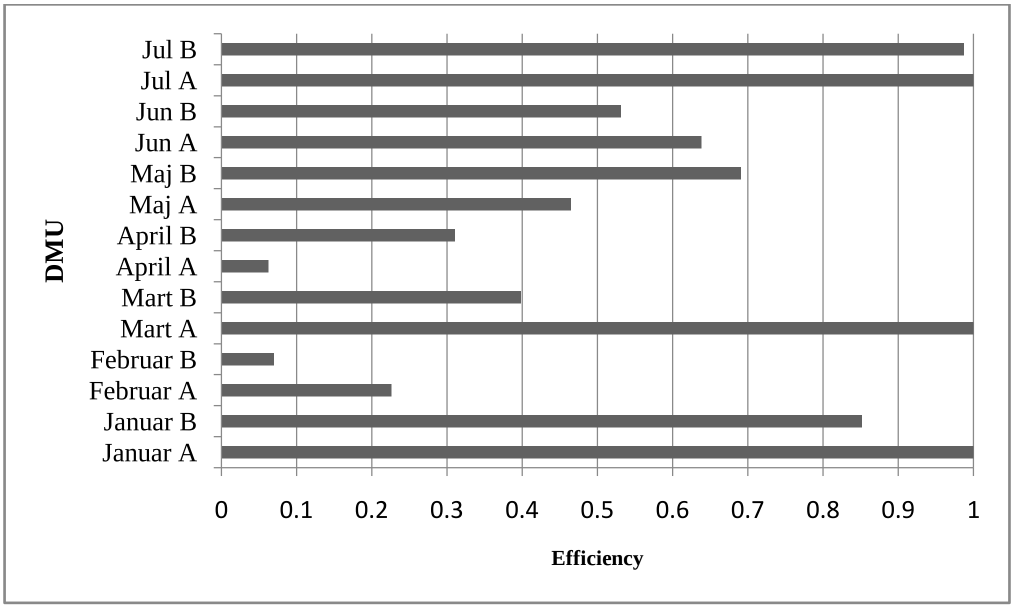

Figure 1. Technical efficiency

5. Conclusion

By application of output-oriented DEA model the technical efficiency of production process by production shifts in automotive industry was measured in the paper. The results have shown that the production in shift A, during three months was technically efficient, i.e. that the utilization of identified production resources was 100%. The limitations of work reflect on relatively small number of primarily input variables, so it can be said that the suggested DEA model is not reflected completely in all characteristics of the observed production process. In that context, future research in automotive industry, by application of DEA model, could be realized by widening the model by introducing new variables, paying attention to discriminatory power of a model, then by introducing additional limitations, in order to remove potential risk of ignoring the influence of the production of certain variables or measuring individual technical efficiency, as well as the efficiency of production process in certain time period or by product variants, by which the conditions for comprehensive analysis and evaluation of production performances. In this way formal framework for optimization of decision making process of production management would be created.

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