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## MORI-TANAKA METHOD IN CHARACTERISATION OF COMPOSITE STRUCTURES

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**Abstract:** *This paper describes the theory of the dependence of the tribological characteristics of the material (coefficient of friction and wear) and the mechanical properties of the material (Poisson's ratio and modulus of elasticity). A special focus is placed on the characteristics of composite materials which are difficult to predict by mechanical and tribological characteristics. Relationship between effective values of friction coefficient and wear and modulus of elasticity is given for composites. Mori-Tanaka homogenization method is applied in calculation of modulus of elasticity and Poisson's ratio, and their dependencies, for composites with spherical particles.*

**Keywords:** *Mori-Tanaka method, homogenisation, composites, friction coefficient*

### 1. INTRODUCTION

It is well known that tribological characteristics of materials such as friction coefficient and wear of materials are very important for tribological applications. When it comes to composite materials, the situation undermines the inhomogeneity of the material itself. As we know, composite materials consist of two or more materials. Each of them has separate material characteristics, and effective material characteristics can be determined by different experimental techniques (tensile test, pressure, indentation) [1]. When creating the composites themselves, it is necessary to take into account the material characteristics of the new material. By predicting the material characteristics of the composite, we can further predict the tribological characteristics. Determination of material properties requires

the use of knowledge in mathematics, mechanics, and material science [2]. For the purpose of predicting the material characteristics of composites, different methods have been studied, such as Mori-Tanaka method, Hashin-Shtrikman bounds, Lielens method, Self-consistent scheme [3], as well as fem modeling [4].

The characterization of composite materials is a very complex area. Particularly complex is the homogenization of hybrid composites that comprise several materials with different material characteristics. Depending on the nature of the composite materials, there may be materials with a matrix and various types of inclusion, such as spherical particles, fiber particles, as well as multilayer materials and woven fiber composites. Mori-Tanaka method represents fast and efficient homogenisation method and provides reliable material

properties. By using this method, it is possible to study composite materials by determining the parameters, such as volume fraction of the matrix and inclusion, shape and size of inclusion, types of layers and features in multilayer composites. It is realised in order to obtain the most favorable characteristics of the material from different aspects. Such methods are particularly important in the prediction of material parameters and mechanical characteristics of new micro composite materials. This paper deals with a relationship between the tribological characteristics of the material (friction coefficient and wear) and the mechanical properties of the material (Poisson's ratio and modulus of elasticity).

## 2. FRICTION COEFFICIENT AND WEAR OF COMPOSITES

The friction coefficient is related to two components: adhesion (due to the force of molecular interaction of contact surfaces) and deformation component. By increasing the modulus of elasticity, the surface strength of the material increases, and consequently the deformation component of the friction coefficient decreases. By increasing the material's elasticity modulus, the contact surface of the sample and the element with which this material is in contact decreases, and thus reduces the adhesive force of attraction between interactive surfaces. As a result, the friction coefficient will decrease.

The functionally graded material can be in the form or in the form of a mono component material with an elastic gradient, caused, for example, by an uneven distribution of residual tensions. Both types of sorted material are widely used in tribo-compounds.

### 2.1 The law of mixtures in the calculation of the friction coefficient and wear

In case of fibers as reinforcements, the friction coefficient  $\mu$  can be given by [5]:

$$\mu = \frac{F}{W} = \frac{F_f + F_m}{W_f + W_m} \quad (1)$$

where  $W$  is normal load,  $F$  is tangential force, suffixes  $f$  and  $m$  denote fiber and matrix, respectively. The following equations are hypothesized:

$$A_f = V_f A \quad (2)$$

$$A_m = V_m A = 1 - V_f A \quad (3)$$

where  $A$  is the nominal area of contact,  $V_f$  and  $V_m$  are the volume fractions of the fibers and matrix, respectively. Under certain conditions, (e.g. in absence of fibers peel-off), shear strain  $\gamma A_f$  is equal to  $\gamma A_m$ :

$$\gamma A_f = \gamma A_m \quad (4)$$

If  $GA_f$  and  $GA_m$ , the moduli of rigidity of materials underneath the contacting surface, are equal, then the shear stress  $\tau$  becomes constant, that is:

$$\tau_f = \frac{F_f}{A_f} = \tau_m = \frac{F_m}{A_m} \quad (5)$$

where:

$$F_f = V_f F \quad (6)$$

$$F_m = V_m F \quad (7)$$

By using the relationship  $W = W_f + W_m$ , the following equations can be obtained:

$$\frac{F}{\mu} = \frac{F_f}{\mu_f} + \frac{F_m}{\mu_m} \quad (8)$$

$$\frac{1}{\mu} = V_f \frac{1}{\mu_f} + V_m \frac{1}{\mu_m} \quad (9)$$

From this equation we are able to calculate the effective friction coefficient  $\mu$  of composite material strengthened by the fibers when the friction coefficients of the fibers  $\mu_f$ , and of the matrix,  $\mu_m$ , are given.

When the composite material strengthened by the fibers is hybrid-reinforced with two fibers  $f_1$  and  $f_2$ , the law of mixtures in the calculation of the friction coefficient is given by [5]:

$$\frac{1}{\mu} = V_{f1} \frac{1}{\mu_{f1}} + V_{f2} \frac{1}{\mu_{f2}} + V_m \frac{1}{\mu_m} \quad (10)$$

$$V_{f1} + V_{f2} + V_m = 1 \quad (11)$$

The wear volume ( $W$ ) is proportional to the contact area ( $A$ ) and the sliding distance ( $S$ ), as in:

$$W = A * S * C \quad (12)$$

where  $C$  is a non-dimensional wear coefficient defined as a function of  $CS$  and a non-dimensional contact stress ( $\frac{P}{E}$ ), where  $E$  is the elastic modulus of the material:

$$C = fun(CS, \frac{P}{E}) \quad (13)$$

Cross shear ratio ( $CS$ ) was defined as the frictional work component perpendicular to the principal molecular orientation direction ( $E_{cross-shear}$ ), divided by the total frictional work ( $E_{total}$ ) [6]:

$$CS = \frac{E_{cross-shear}}{E_{total}} \quad (14)$$

### 3. FINITE ELEMENT METHOD IN CHARACTERISATION OF MECHANICAL PROPERTIES FOR COMPOSITE MATERIALS

The finite element method (FEM) can efficiently support simulation of experimental testing of composite structures. When simulating the compressive test by using the finite element method, one can clearly conclude that the friction that occurs between the surface of the sample and the compression plate has a slight influence on the deformation in the direction of the compression plate operation. From this, it can be concluded that the experimentally determined modulus of elasticity is valid.

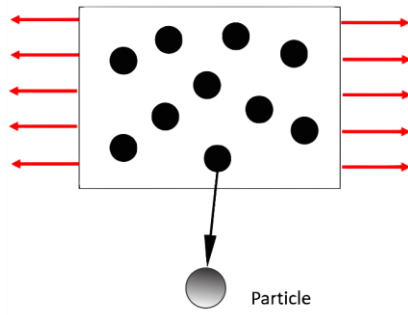
Unlike the previous, the friction affects the lateral deformation of the sample on the contact surface. The greater the friction coefficient, the smaller Poisson coefficient. In order for the test results to be relevant, the contact surfaces should be polished or lubricated [7].

#### 3.1 Determination of the material properties of composite (Mori-Tanaka method)

Micro-mechanics of materials is the analysis of composite or heterogeneous

materials at the level of the individual constituents, commonly used for nano and biomedical materials. As a multidisciplinary area, it covers mechanical, electrical, and in general thermodynamic behavior. Composites comprise different phases throughout the structure, and these can be of completely different mechanical and physical properties. Microstructural properties of such materials can be modeled with isotropic behavior, but in reality they exhibit anisotropy due to different relevant properties within one structure. Anisotropic material models are available for linear elasticity. In the nonlinear regime, the modeling is often restricted to orthotropic material models, which do not capture the physics for all heterogeneous materials. Micro-mechanics goal is to predict the anisotropic response of the heterogeneous material based on the geometries and properties of the individual phases, aiming at homogenization. Micro-mechanical analysis is the discipline that requires the theoretical framework and the related tools. There are two different approaches or methods in micro-mechanics: analytical and numerical approaches. Analytical methods of continuum micro-mechanics are Voigt method, Reuss method, Strength of Materials (SOM), Vanishing Fiber Diameter (VFD), Composite Cylinder Assemblage (CCA), Hashin-Shtrikman Bounds, Self-Consistent Scheme and Mori-Tanaka method. Numerical approaches to continuum micro-mechanics are Finite Element Analysis (FEA), Mechanics of Structure Genome (MSG), Generalized Method of Cells (GMC), and Fast Fourier Transforms (FFT) [8], [9].

For homogenization of the composite and determination of effective mechanical characteristics by the Mori-Tanaka method, it is necessary to set the input parameters that characterize the matrix and inclusion. These are Poisson's' ratio, modulus of elasticity and volume fraction for each individual. The obtained effective characteristics of the material can be validated by using standard mechanical tests, like tensile test (Fig. 1), pressure test, indentation test, etc.



**Figure 1.** Composite structure – Matrix with spherical inclusion loaded with tension

Based on Eshelby solution fundamental results in ellipsoidal inclusion, the total strain induced by the appearance of Eigenstrain is uniform. The uniform strain field inside inclusion can be expressed as a function of the Eigenstrain per:

$$\varepsilon = S\varepsilon^* \quad (15)$$

Where S is fourth order Eshelby tensor. Eshelby tensor depends on the material properties and the shape of the inclusion (cylindrical or spherical). Analytical expressions can be found for isotropic linear materials for some specific shapes. Fourth order Eshelby tensor is given by:

$$S = \begin{bmatrix} S_{1111} & S_{1122} & S_{1133} & 0 & 0 & 0 \\ S_{2211} & S_{2222} & S_{2233} & 0 & 0 & 0 \\ S_{3311} & S_{3322} & S_{3333} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2S_{1212} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2S_{1313} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2S_{2323} \end{bmatrix} \quad (16)$$

The effective modulus requires the definition of the strain concentration tensor **A**. In order to calculate the effective modulus the Mori-Tanaka approximation has been chosen. The expression of the strain concentration tensors are identified as:

$$A_r = T_r \left( \sum_{r=0}^N C_r T_r \right)^{-1} \quad (17)$$

$$A_0 = \frac{1}{C_0} \left( I - \sum_{r=1}^N C_r T_r \right)^{-1} \quad (18)$$

The effective stiffness tensor is obtained from the expression of the concentration tensors as:

$$L = \sum_{r=0}^N C_r L_r A_r \quad (19)$$

Taking an isotropic stiffness tensor it is possible to obtain effective values respectively:

$$\lambda_{ef} = L_{12} \quad (20)$$

$$\mu_{ef} = \frac{1}{2} (L_{11} - L_{12}) \quad (21)$$

$$E_{ef} = \frac{\mu_{ef} (3\lambda_{ef} + 2\mu_{ef})}{\lambda_{ef} + \mu_{ef}} \quad (22)$$

$$\nu_{ef} = \frac{\lambda_{ef}}{2(\lambda_{ef} + \mu_{ef})} \quad (23)$$

$$\kappa_{ef} = \frac{3\lambda_{ef} + 2\mu_{ef}}{3} \quad (24)$$

The aim of this study was to obtain effective properties of the fiber composite as a function of Young's modulus varying from 7 to 110 GPa using simplified Mori-Tanaka method, whereas the structure is shown in Fig. 1. We applied simplified Mori-Tanaka method to obtain analytical dependences between Young modulus of elasticity, shear modulus and Poisson's ratio and relationship to matrix and inclusion properties, and results are given in Figs 2-5.

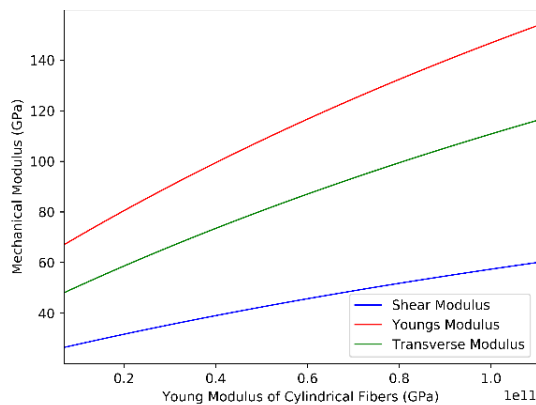
**Table 1.** Material properties of composite material

Material constants	E (GPa)	v (-)	c (-)
Matrix	207e9	0.3	0.34
Inclusion	from 7e9 to 110e9	0.26	1 - c <sub>m</sub>

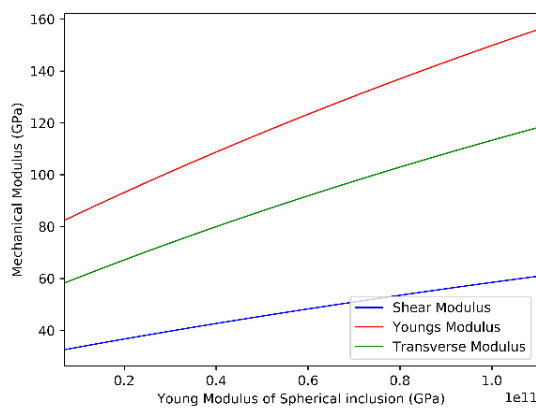
Numerical calculations were realised for composites with spherical and cylindrical particles and fibers. Obtained results are homogenized values, or effective values, of Young modulus and Poisson's coefficient for composite structure.

It can be seen that there is significant difference between values of Young modulus and Poisson's ration for matrix material, reinforcement material and the composite structure, indicating that effective values must be included in characterisation of composites. These results can be further used to predict the tribological characteristics of the composite material because the coefficient of friction and wear depends

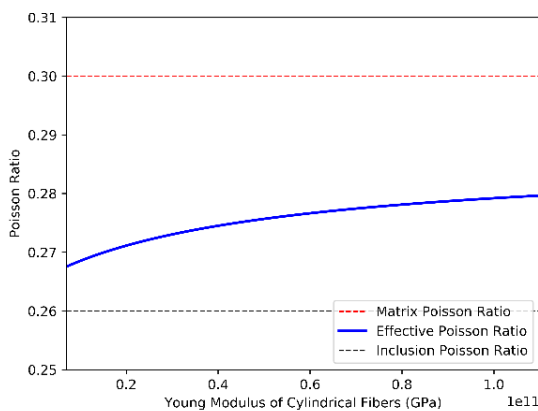
directly on material parameters such provided by Mori-Tanaka method and numerical solving of the homogenization problem of composite materials.



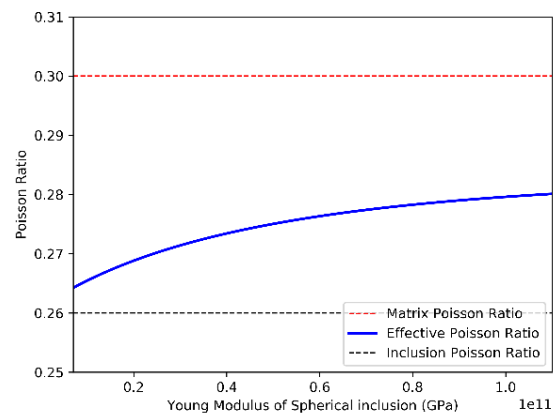
**Figure 2.** Effective mechanical modulus of composite (Young modulus, shear modulus and transverse modulus) as the function of Young modulus of cylindrical fibers as reinforcements



**Figure 3.** Effective mechanical modulus of composite (Young modulus, shear modulus and transverse modulus) as the function of Young modulus of spherical particles as reinforcements



**Figure 4.** Poisson's ratio as the function of Young modulus of cylindrical fibers as reinforcements



**Figure 5.** Poisson's ratio as the function of Young modulus of spherical particles as reinforcements

#### 4. CONCLUSION

Material parameters such as the Young modulus of elasticity and the Poisson's ratio have a direct relationship with tribological properties. In order to predict the behaviour of coefficient of friction and wear of composite material, it is necessary to determine effective material characteristics. Mori – Tanaka method, together with finite element analysis, is efficient approach that can provide necessary properties, such as Young modulus of elasticity over range of loading. These effective values can be further used in characterisation of tribological behaviour of composites.

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